PLAIN TRIGONOMETRY

RENDERED EASY AND FAMILIAR,

BY CALCULATIONS IN ARITHMETICK ONLY:

WITH ITS

APPLICATION AND USE

IN ASCERTAINING

All Kinds of HEIGHTS, DEPTHS, and DISTANCES,

IN

The HEAVENS, as well as on the EARTH and SEAS;

WHETHER OF

TOWERS, FORTS, TREES, PYRAMIDS, COLUMNS, WELLS, SHIPS, HILLS, CLOUDS, THUNDER AND LIGHTNING, ATMOSPHERE, SUN, MOON, PLANETS, MOUNTAINS IN THE MOON, SHADOWS OF EARTH AND MOON, BEGINNING AND END OF ECLIPSES, &c.

IN WHICH IS ALSO SHEWN,

A Curious TRIGONOMETRICAL METHOD of difcovering the Places where BEES HIVE in LARGE WOODS, in Order to obtain, more readily, the SALUTARY PRODUCE of thofe LITTLE INSECTS.

By the Rev. R. TURNER, of *Magdalen Hall, Oxford*;

Author of *The View of the Earth*;—*View of the Heavens*;—*Syftem of Gauging*;—and *Chronologer Perpetual*:

Rector of COMBERTON, Vicar of ELMLEY, Minifter of STOULTON, and Chaplain to the Right Honourable the COUNTESS DOWAGER of WIGTON.

Cuncta Trigonus *habet, fatagitquedocta* Mathefis,
Ille *aperit claufum, quicquid* Olympus *habet.*

Within the grand Triangle *lies unveil'd,*
What Sages *fought for, and what* Heaven *conceal'd.*

LONDON:

PRINTED FOR S. CROWDER, IN PATER-NOSTER ROW. 1778.

TO THOSE

GENTLEMEN,

WHOSE GENIUS MAY INCLINE, OR EMPLOYMENT LEAD THEM TO THE STUDY OF THE

MATHEMATICKS.

GENTLEMEN,

TRIGONOMETRY has always been look'd on as one of the most useful Branches of *Mathematical* Learning. *Navigation*, *Surveying*, *Astronomy*, &c. stand wholly upon this *Basis*. But the common Method of answering these Problems by large TABLES of *Sines*, *Tangents*, and *Secants*, renders it not only expensive by the Purchase of them; but often precarious in the Solution, through Mistakes of the Press. I have therefore, for the Use of the *Young Mathematician*, (from a Consideration of what has been published on this curious Subject) composed the present *System*, by which any of the *Cases* in *Right* or *Oblique Plain Triangles* may be answered on the *Spot*, by an easy Calculation in *Arithmetick* only *. The great Advantages resulting from this Method to *Gentlemen* in the *Army* or *Navy*, as well as to Those in their *private Studies* at *Home*, must immediately appear; as it will be found to answer the most necessary Problems as *expeditiously* as *Logarithms*, (oftentimes more so;) and at the same Time wholly deliver you from those *voluminous Tables* and the *inartificial Fatigues* of carrying them always with you.—Should this little Treatise be so happy as to meet your Approbation, it will give a particular Pleasure to,

Your most humble Servant,

The AUTHOR.

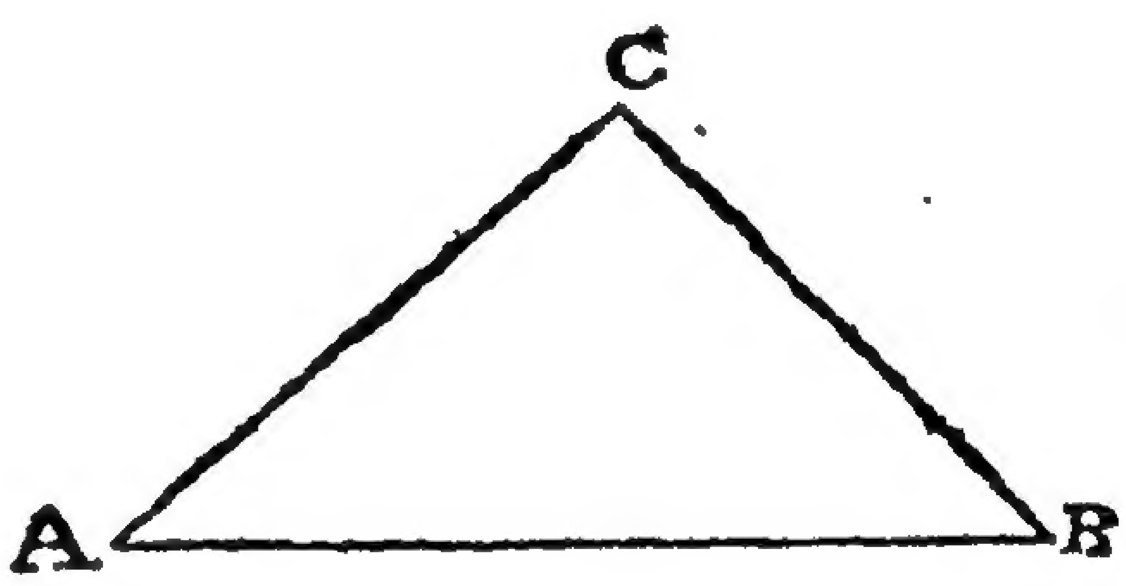
* It is wished that some Gentleman would undertake the Publication of *Spherical Trigonometry* in a similar Manner.

PLAIN

TRIGONOMETRY.

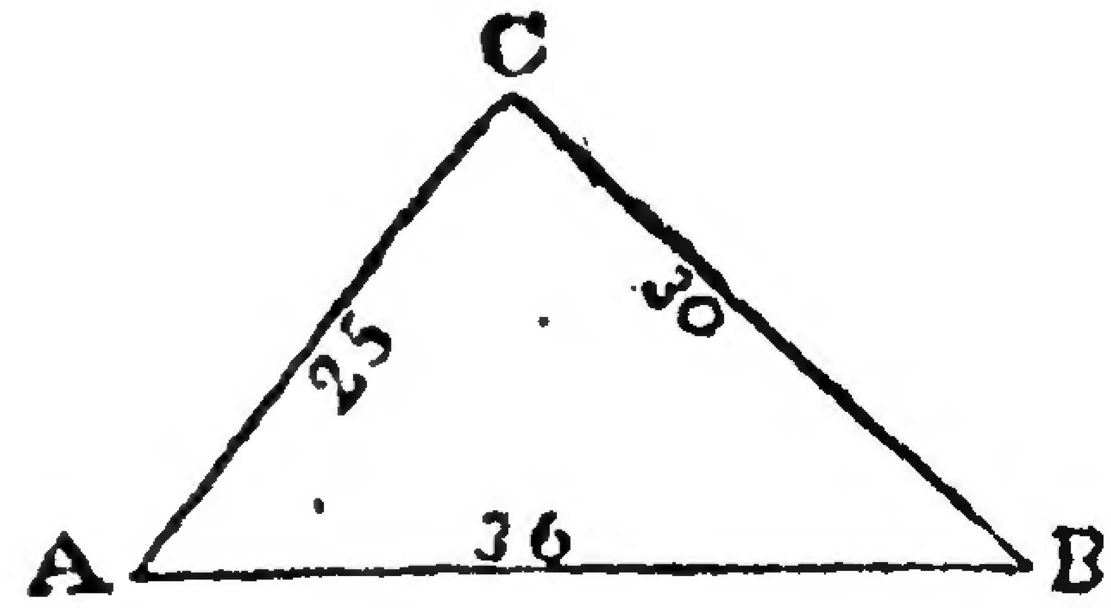
TRIGONOMETRY is that Part of *Mathematicks*, which is employed in calculating the *Sides* and finding the *Angles* of any *Triangle* required; it is of the greateſt *Uſe*, as nothing in *Navigation, Aſtronomy*, &c. can be done without it; and depends on the Knowledge of the following Obſervations, or Properties of that Figure.

(1ſt.) Every Triangle conſiſts of Six Parts; that is, of *Three Sides* and *Three Angles*, as in the Figure ABC; the Three Sides are, AB, AC, CB, and the Three Angles, A, B, C.



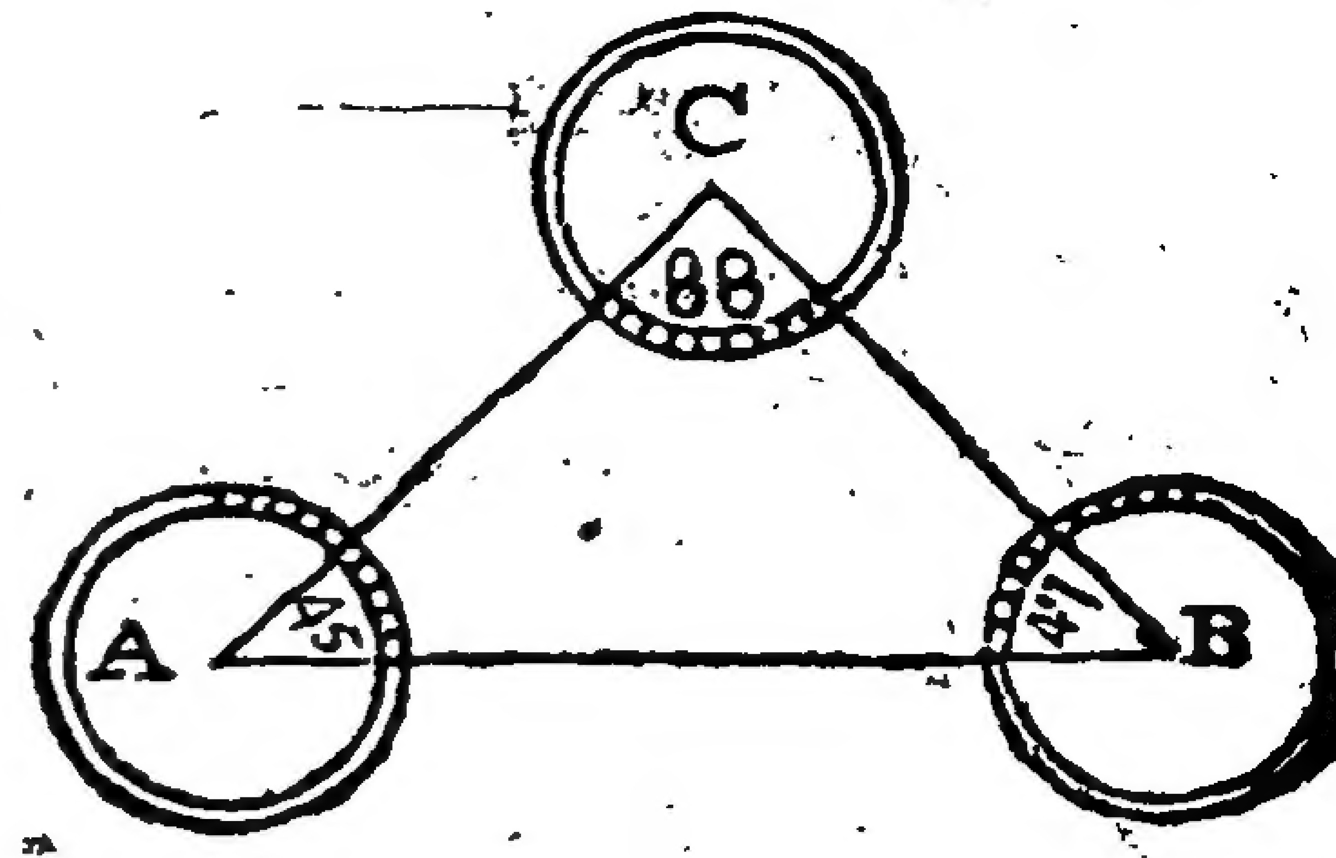
NOTE. Sometimes an *Angle* is expreſſed by *Three* Letters; in that Caſe, the *Middle* Letter denotes the *Angular Point*. Thus, ABC expreſſes the Angle B; BAC the Angle A; and ACB the Angle C.

(2d.) The Sides of all plain Triangles are meaſured by a Line of equal *Parts*, as of *Inches*,—*Feet*,—*Yards*,—or *Leagues*.



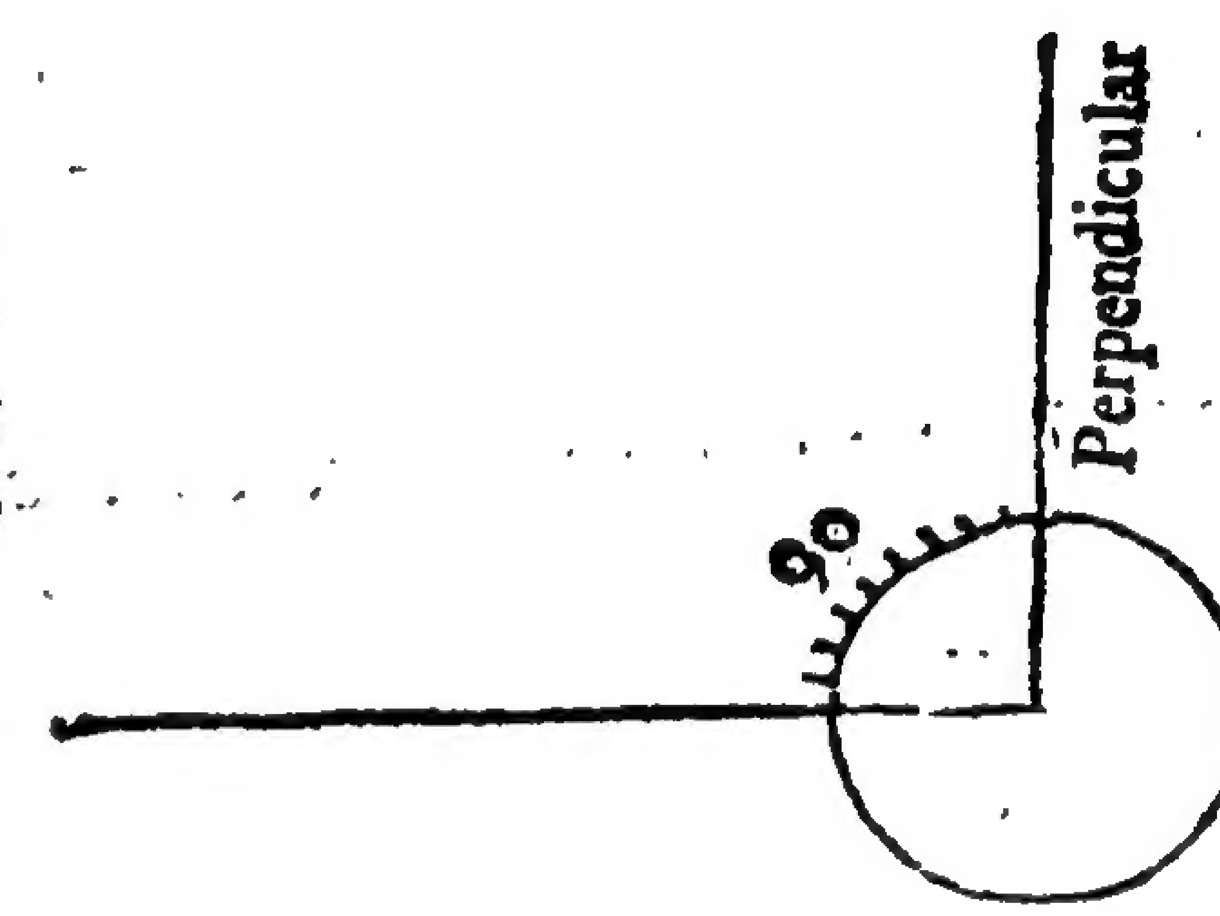
Thus, The Side AB is 36 Leagues.---The Side AC 25 Leagues.--- And the Side BC is 30 Leagues.

(3d.) The

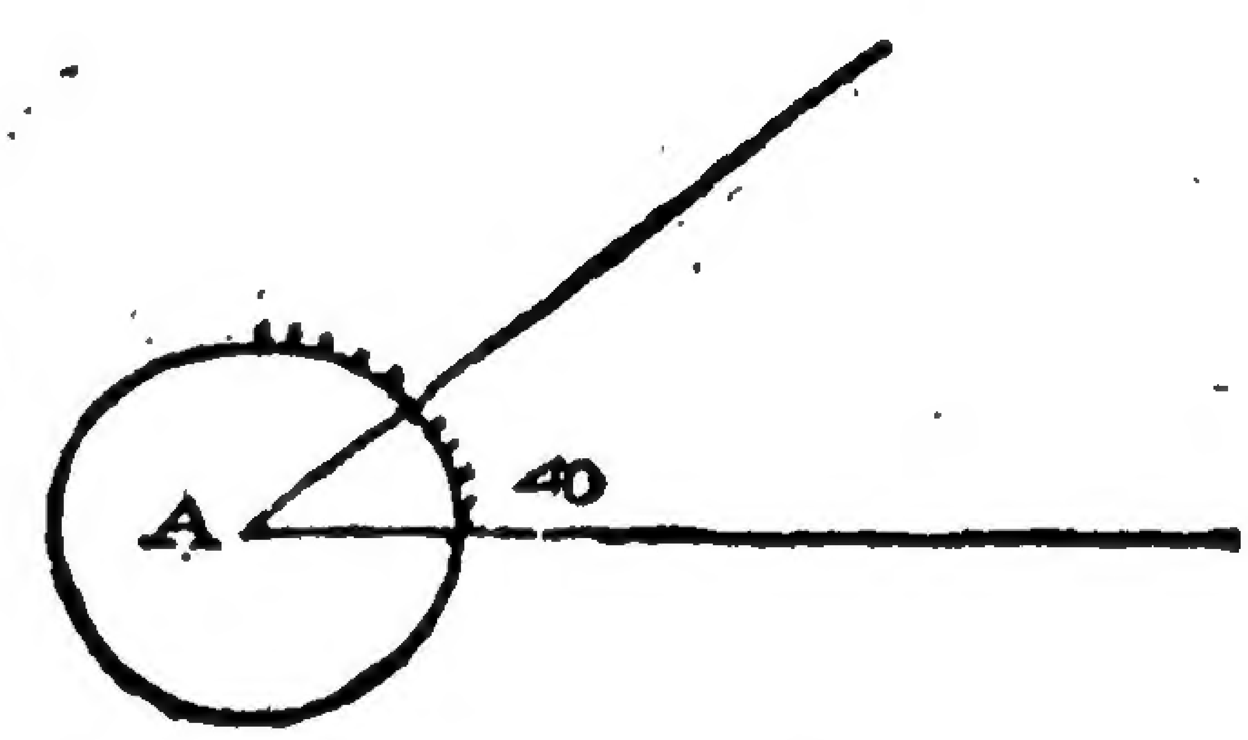


(3d.) The *Angles* are meaſured by the *Arch* of a *Circle* deſcribed upon the Angular Point, and contained between the Two Legs that form the Angle.

Note. Every Circle is divided into 360 equal Parts, called *Degrees*; each of which is divided into 60 more, called *Minutes*: And the Number of Degrees contained between the Two Legs, that conſtitute the Angle, is the *Meaſure* of that Angle. Thus, The Angle A is 45 *Degrees*.---The Angle B 47.---The Angle C 88.

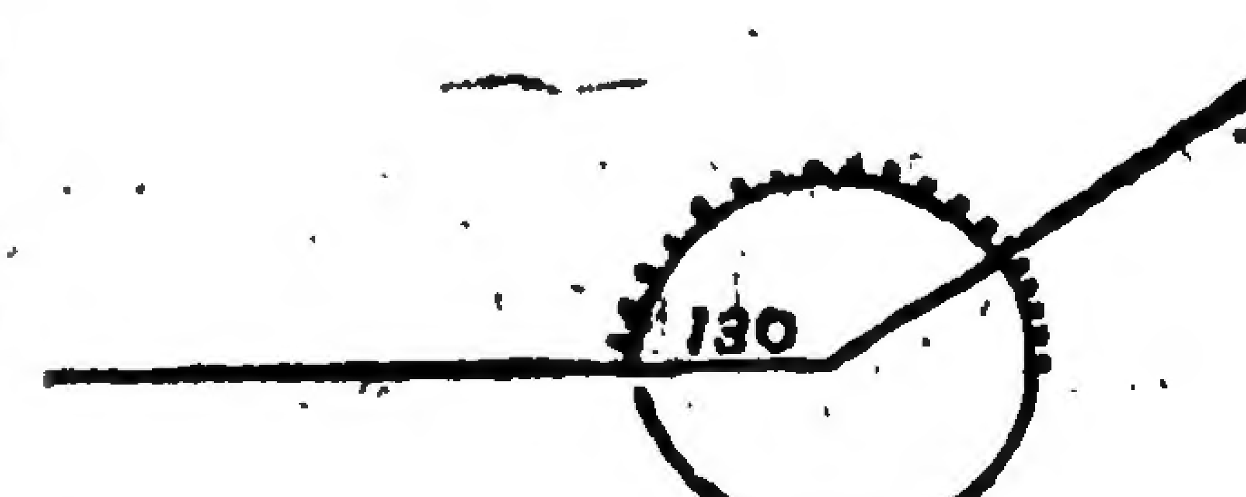


(4th.) If the Arch of a Circle intercepted between the Two Legs be exactly 90 Degrees, the Angle is called a *Right Angle*, and the Legs are *perpendicular* to one another.



(5th.) If the Arch of the Circle be *leſs* than 90 Degrees, the Angle is ſaid to be *Acute*.

Note. What an *Acute* Angle wants of 90 Degrees, is called the *Complement* of that Angle. Thus, Suppoſe the Angle A was 40 Degrees; then its Complement is 50 Degrees; for 40 added to 50 make 90, as obſerved before.

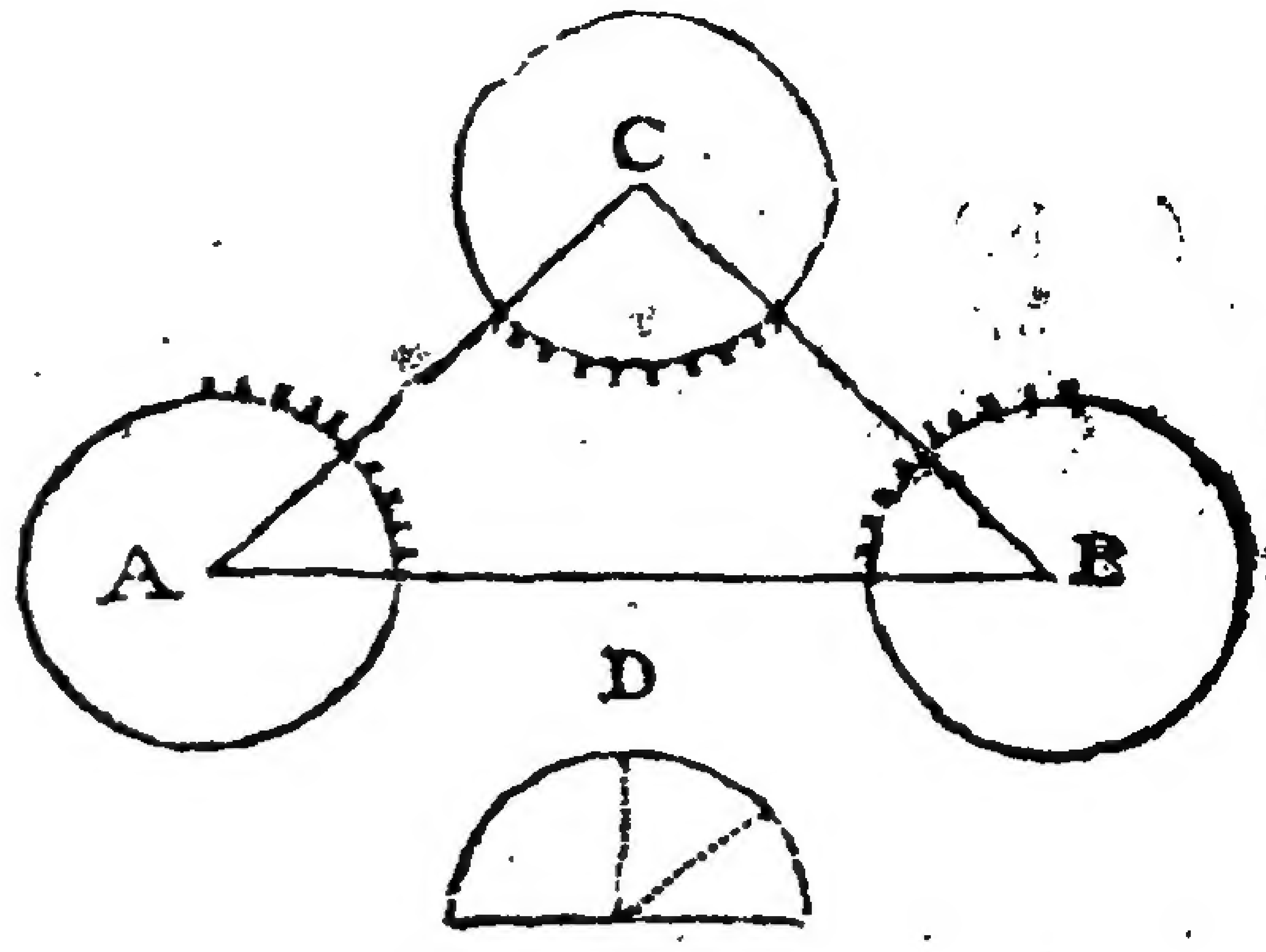


(6th.) If the Arch of a Circle be *more* than 90 Degrees, the Angle is ſaid to be *Obtuſe*; and ſo continues to 180 Degrees,* where the Angle vaniſhes, the Lines becoming *Strait*.

Note. What an *Obtuſe* Angle wants of 180 Degrees, is alſo called the *Complement of that Angle to a Semicircle*.

(7th.) The

(7th.) The Three Angles of every plain Triangle, being taken together, make 180 Degrees (equal to a Semicircle), and this they always do, let the Triangle be drawn however you please.

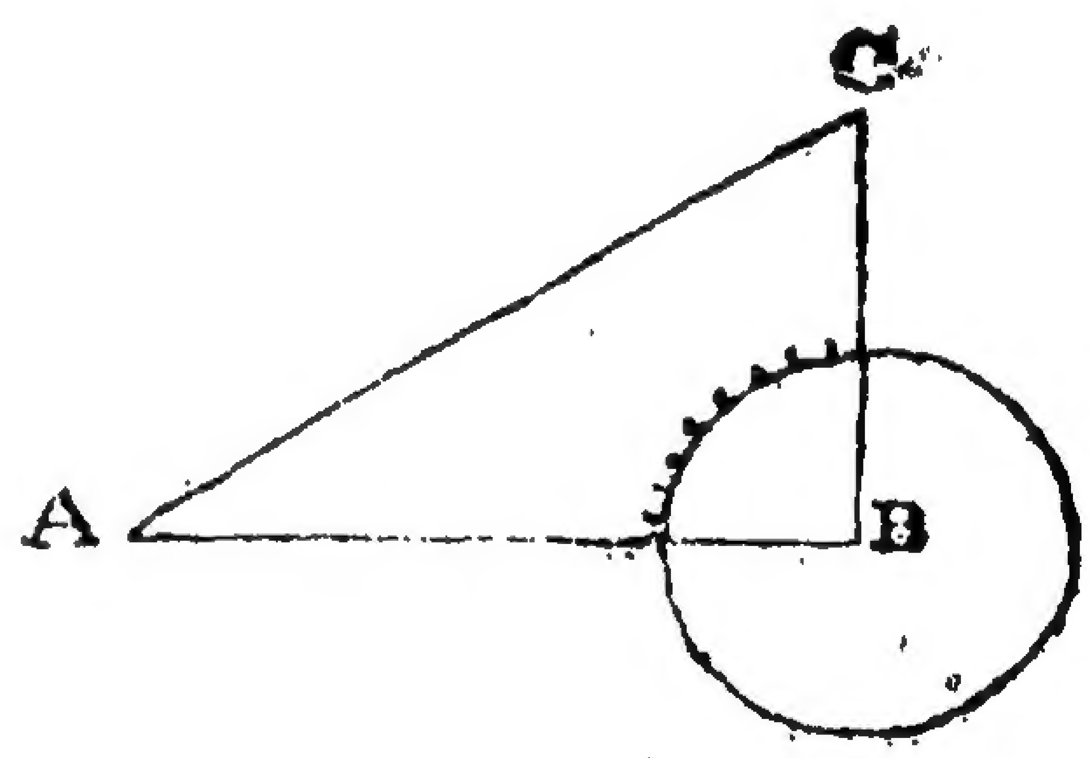


Thus, If the *Semicircle* D be drawn with the same *Radius*, or opening of the Dividers, as the little Circles on the Angles A, B, C, are; you will find, by taking off the several Arches, and applying them to the Semicircle, that they will just fill it up, and thereby make 180 Degrees; because every Semicircle contains that Number of Degrees.

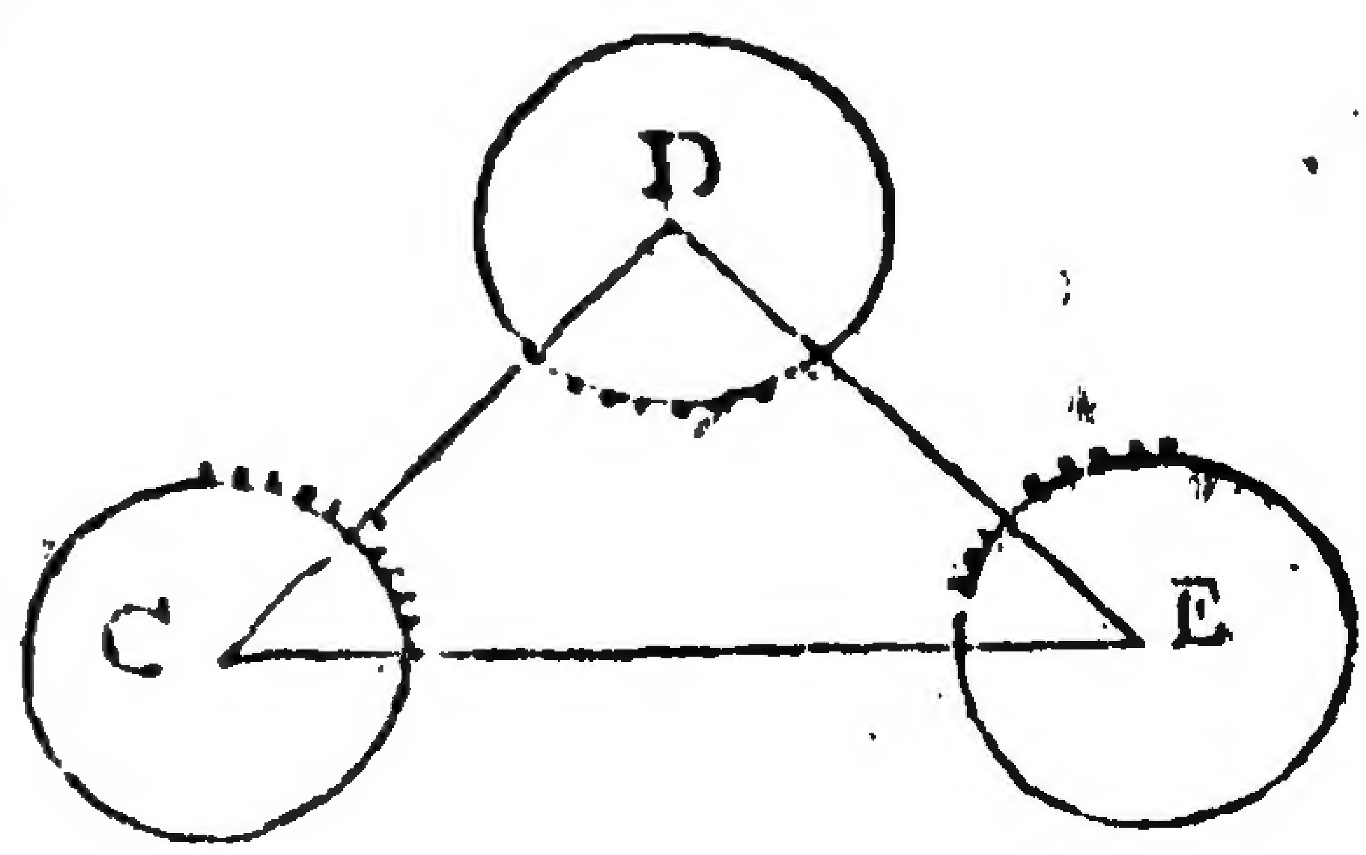
Hence it is evident, that if one Angle be a *Right One*, the other Two will be *Acute*; and taken together, be equal to one *Right Angle*, or just 90 Degrees.

Hence also, if *Two* Angles of any Triangle are known, the *Third* is easily found, being only the *Degrees* the other Two Angles want of 180.

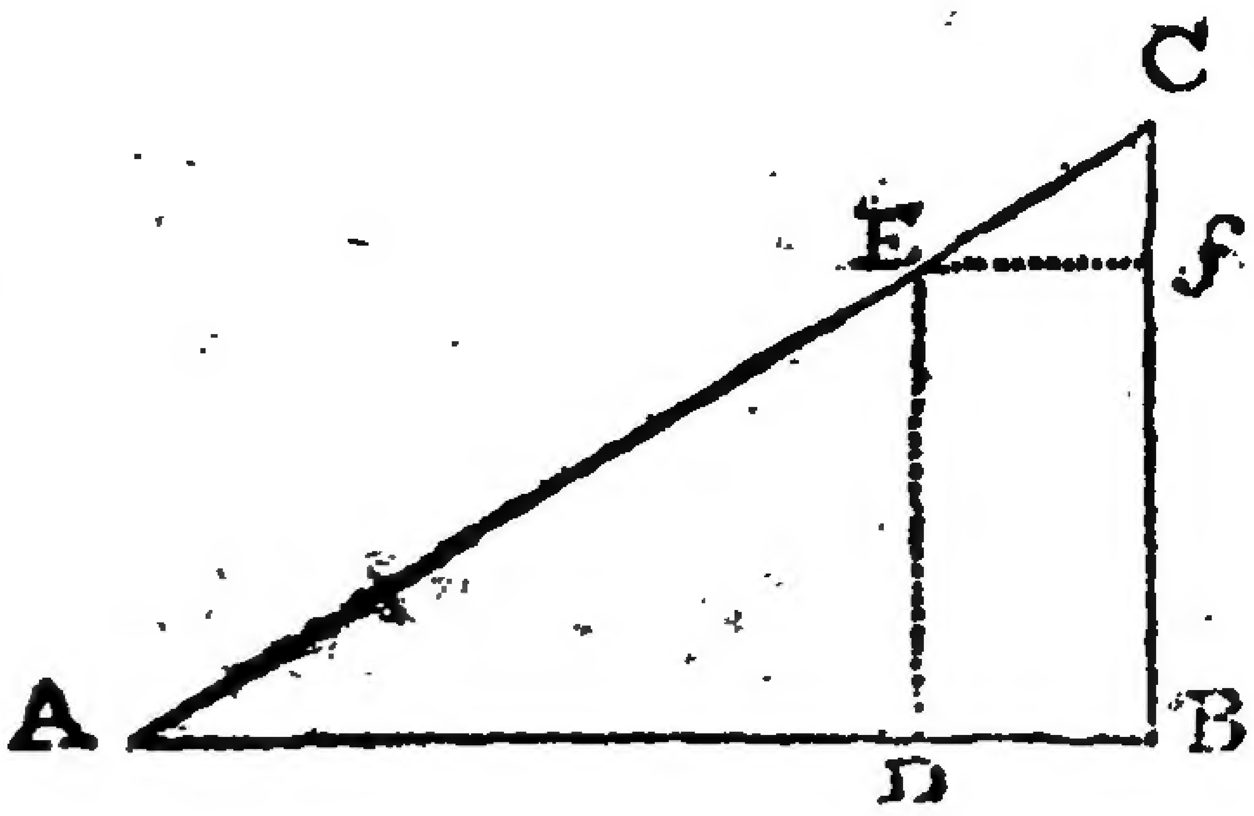
(8th.) If a *Triangle* has one *Right Angle*, it is called a *Right Angled Triangle*: Thus ABC is a Right Angle Triangle, Right-angled at B.---In all Right Angle Triangles, the longest Leg is called the *Hypothenuse*; --- the Leg on which it stands, the *Base*;---and the other Leg, the *Perpendicular*.



(9th.) If neither of the Angles is a Right One, then it is called an *Oblique Angled Triangle*; as the Triangle CDE is an *Oblique Triangle*.



(10th.) If the Angles of one Triangle are equal to the Angles of another Triangle, the Sides of the former are proportioned to the Sides of the latter.



Thus, If in the Triangle ABC, you draw the Line ED parallel to CB, the ſmaller Triangle ADE will be *ſimilar to*, i. e. will have the ſame Angles with the larger Triangle ABC.---- It will therefore always hold, as

AB : AD : : BC : DE.
Or, as AB : BC : : AD : DE.
Or, as AD : DE : : AB : AC.
Or, as AD : DE : : E*ſ* : *ſ*C, *&c.*

(11th.) In all Triangles the *greateſt Side* is oppoſite to the *greateſt Angle*; and on the contrary, the *greateſt Angle* is oppoſite the *greateſt Side*.---If Two *Sides* are *equal*, the oppoſite *Angles* are *equal*.---If all the Sides are equal, then all the Angles are equal to each other.

(12th.) In all Triangles, every Side is in proportion to its oppoſite Angle, and every Angle to its oppoſite Side: And further, as the Angle oppoſite to one Side, is to the Angle oppoſite the other Side, ſo are the Sides themſelves one to another; and the contrary, the Sides to the Angles.

Every *Triangle*, as I obſerved before, conſiſts of Six Parts---*Three Sides* and *Three Angles*. If any Three of the Six Parts (excepting the Three Angles) are given, any one, or every one of the reſt may be found, without the painful Deductions and voluminous Tables of *Logarithms*, *Sines*, *Tangents*, and *Secants*, by the following *Rules* and *Axioms* *.

* This *Method* will be found as exact, as that by the *Logarithms*, if you carry on the *Operation* to *Three* or *Four Decimal* Places; but for common Purpoſes, *One* or *Two Decimal* Places will be near enough. You muſt alſo remember to reduce the *Minutes* (and *Seconds*) of the Angles to *Decimals* of a Degree, which is eaſily done, by allowing One Tenth for every Six Minutes.——Or you may turn the Minutes into Decimals thus: As 60, the Minutes in one Degree, : are to the Minutes given, : : ſo are 10, 100, 1000, &c. to the *Decimal* required.

The foregoing Properties of a Triangle are ſo manifeſt, that they ſtand in need of no further Illuſtration or *Demonſtration*.

Of

Of Right Angled TRIANGLES.

THERE are generally reckoned by Writers on this Subject Seven Cases; but by this Method they are all reduced to Four; the Solutions of which depend on the following *Axioms*.

AXIOM I. Divide 4 Times the Square of the Complement of the Angle, whose opposite Side is either given or sought, by 300 added to 3 Times the said Complement; this Quotient added to the said Angle, will give you an *Artificial Number*, called sometimes the *Natural Radius* *, which will ever bear the same Proportion to the Hypothenuse, as that Angle bears to its Side.——In Angles under 45 Degrees, the *Artificial Number* may be found thus: Divide 3 Times the Square of the Angle itself, whose opposite Side is given or sought, by 1000; the Quotient added to 57.3 †, a fixed Number, that Sum will be the *Artificial Number* required.---*This is to be used, when the* Angles *and a* Side *are given, to find* another Side.

This Axiom is derived from collecting together a few leading Terms of a swift converging Series for determining the Length of the Sine or Co-sine, from the Length of a given Arch. Thus, If x represent the versed Sine, and z the Length of the correspondent Arch, the Radius being r, we shall have $x = \frac{z^2}{2r} - \frac{z^4}{24r^3}$ (see Simpson's Fluxions, p. 500) and, consequently, the Cosine of the Arch $z = r + \frac{z^4}{24r^3} - \frac{z^2}{2r}$. Now if we suppose $r = 1$, the said Co-sine will be $1 + \frac{z^4}{24} - \frac{z^2}{2}$, which gives this rule: "Square the Complement of the given Arch. "and divide it by 2, subtract one Sixth of the Quantity squared from "the Quotient, and the Remainder from the Radius $= 1$. the Resi-"due will be the Sine of the proposed Arch." This Rule will give the Sine true to three Places in Decimals; but the Length of the Arch measuring the Angle must be taken in Terms of the Radius: In Order to remove this Difficulty let the Radius be increased to 57.3 nearly equal to the Radius of a Circle, whose Circumference is 360 equal Parts. Substitute this Value for r in the above Expression, $r - x = r + \frac{z^4}{24r^3} - \frac{z^2}{2r}$, and we shall have $57.3 + \frac{z^4}{24 \times 57.3^3} - \frac{z^2}{2 \times 57.3}$ for the Cosine of the Arch z, which may now be expressed by the Degrees and Decimal Parts, measuring the given Angle.

* The *Natural Radius* is only turning the *Right Angle*, = 90 Degrees, into an *artificial Number*, which shall always bear the same Proportion to the *Hypothenuse*, as the given Angle does to its opposite Leg.

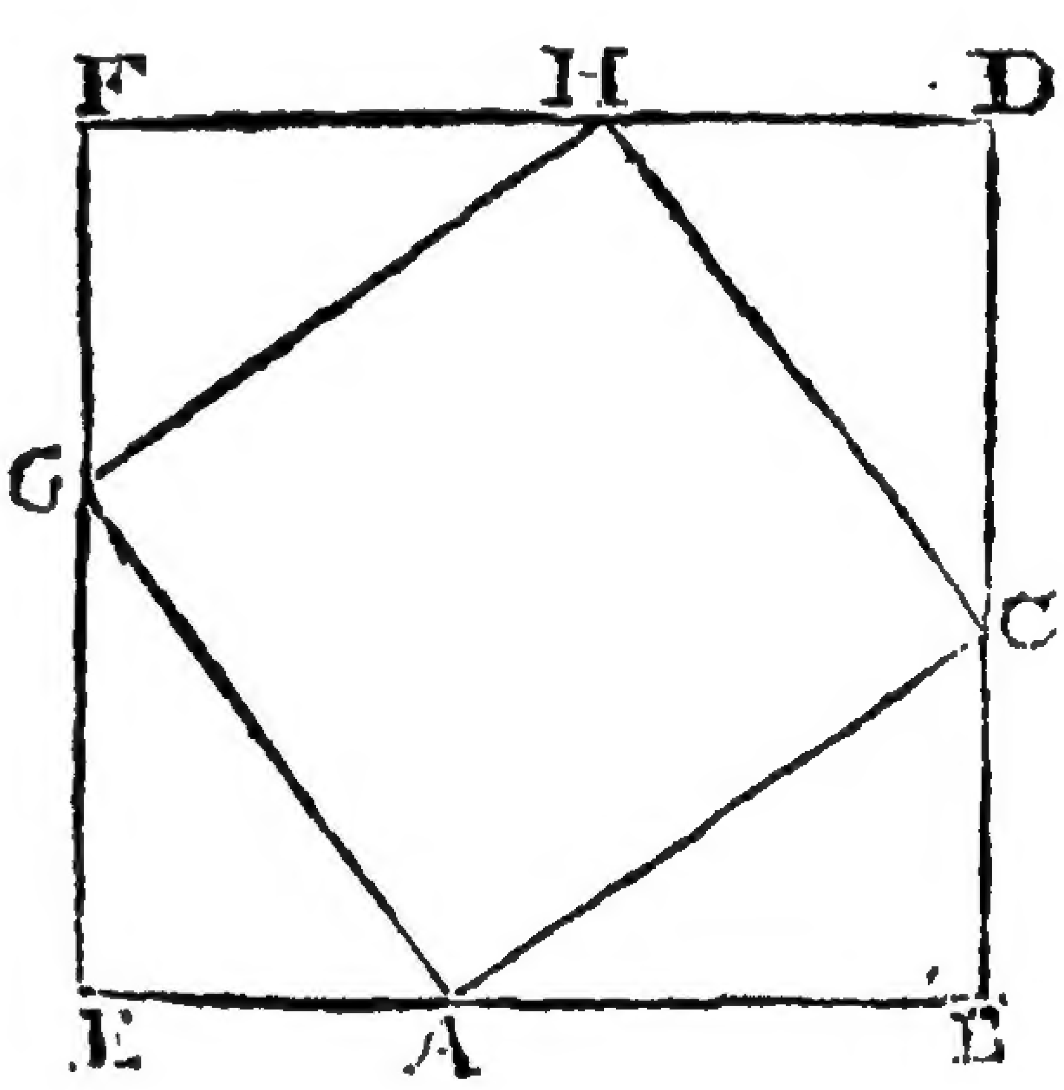
† 57.3 is the *Radius* of a Circle whose Circumference is 360, or more exact, 57 25979.

From

From this Inveſtigation the firſt Part of the above Axiom is eaſily deduced, as will appear by expreſſing that Rule algebraically, in Terms of the Radius 57·3, z being the Co-ſine of the given Angle; for it will ſtand thus: $\frac{4zz}{300+3z} + 90 - z$, the Radius; conſequently, it will be, as $90 - z : \frac{4zz}{300+3z} + 90 - z$: : the Sine of the Angle expreſſed by $90 - z : 57{\cdot}3$, the Radius of a Circle whoſe Circumference is 360 equal Parts. *Q. E. D.*

The ſecond Part of this Axiom may be eaſily inveſtigated in the following Manner: Let $z =$ the Length of any Arch, expreſſed in Terms of r, the Radius, and $y =$ the Sine of that Arch; then will $y = z - \frac{z^3}{2.3r^2} + \&c.$ And if we put $r = 1$, then will $y = z - \frac{z^3}{2.3}$; or, which is the ſame Thing $y = \overline{1 - \frac{z^2}{6}} \times z$, be the Sine of that Arch expreſſed in Terms of the Radius. (See Simpſon's Fluxions, p. 501) But in Order to expreſs it in Terms of the Angle itſelf, we muſt put $r = 57{\cdot}3$, the Radius of a Circle whoſe Circumference is 360 equal Parts. Then will the above Series, when the Angle is leſs than 45 Degrees, give an Expreſſion nearly to $\frac{3zz}{1000} + r = y$, the Rule laid down in the ſecond Part of this Axiom. *Q. E. D.*

AXIOM II. The Square of both the Legs, *i. e.* the Square of the *Baſe* and *Perpendicular* added together, is equal to the Square of the *Hypothenuſe*; whoſe Root is the *Hypothenuſe* itſelf.—*This is made uſe of, when the* Baſe *and* Perpendicular *are given, to find the* Hypothenuſe.



This is the 47th Propoſition of Euclid's 1ſt Book, and may be eaſily demonſtrated in the following Manner.

Let ABC be a Right Angled Triangle, and let the Baſe AB $= a$, the Perpendicular BC $= b$, and the Hypothenuſe AC $= x$; then will $xx = aa + bb$.

DEMONSTRATION.

Continue the Perpendicular BC till CD = AB, and on BD, as a Baſe, draw the Square BDFE. Upon AC, as a Baſe form another Square, as ACHG. Then will the Area of the great Square, BDFE = the Area of the four Triangles ABC, CDH, HFG, GAE + the Area of the Square ACHG. But the Area of each Triangle is $\frac{ab}{2}$; conſequently, the Area of the four Angles is $2\,ab$. Therefore, the Area of the great Square is $xx + 2ab$. But the Area of the great Square is alſo = the Rectangle of $a + b$, or $aa + 2ba + bb$; conſequently, $xx + 2ba = aa + 2ba + bb$. Expunge $2ba$ from both Sides of the Equation, and it will be $xx = aa + bb$. *Q. E. D.*

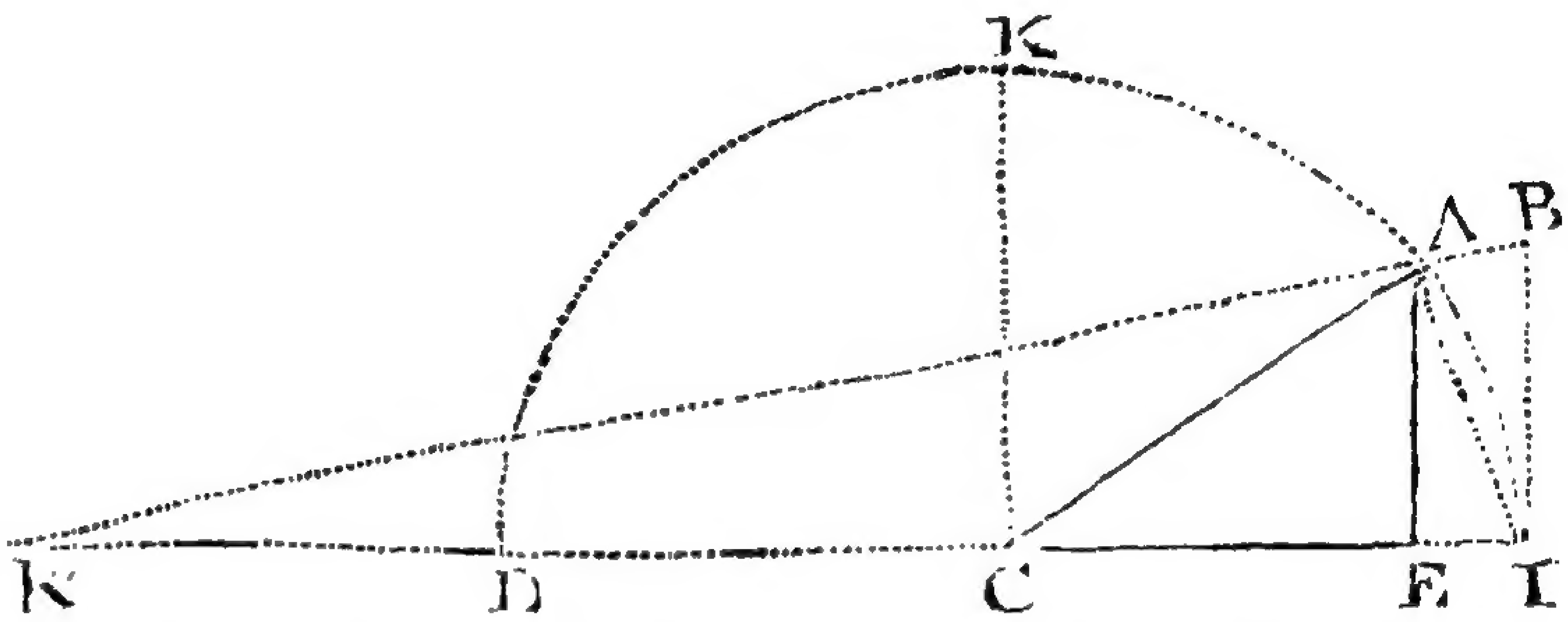
AXIOM

AXIOM III. The *Sum* of the *Hypothenuſe* and *One* of the *Legs* multiplied by their *Difference*, the *Square Root* of that Product will be the *other Leg* required.---*This comes into uſe, when the* Hypothenuſe *and* One Leg *is given, to find the* other Leg.

This is only a Conſequence of the laſt Demonſtration; for it is there proved, that

$$aa + bb = xx. \quad \text{Conſequently } xx - bb = aa.$$
$$\text{But } x + b \times \overline{x - b} = xx - bb. \quad Q.\ E.\ D.$$

AXIOM IV. *Half* the *Longer* of the *Two Legs*, added to the *Hypothenuſe*, is always in Proportion to 86 [*], as the *Shorter Leg* is to its *oppoſite Angle*.---*This is uſeful, when the* Sides *are given, to find the* Angles.



CONSTRUCTION.

Let CAE be the Triangle propoſed. With the Radius CA = the Hypothenuſe, deſcribe the Semicircle IKDC, and, producing the Diameter towards R, make DR = DC, the Radius. Draw the Tangent IB, and the Chord IA. Then will the Chord IA be equal to, or ſufficiently near equal to IB, the Tangent of the Angle ACE; and RI is Triple of the Radius CI = the Hypothenuſe CA.

DEMONSTRATION.

Suppoſing IB = IA; the two Triangles RIB, and REA are ſimilar, becauſe AE is parallel to BI. Therefore it will be (by the 6th Euclid's 4th.)

As RE : EA : : RI : IB = IA. And,
As RE : EA : : RI : IA = IB.

The Chord IA muſt be ſubſtituted for an Arch of a Circle whoſe Circumference = 360 equal Parts, whoſe Half is DKI. But the Radius of a Circle whoſe Circumference is 360, is 57.2 957 = CI, which tripled is 171.8871, or 172 nearly, = RI, by Conſtruction; and RD = DC = CA, the Hypothenuſe: Therefore it will be, as RE = RD + DC + CE, or twice the Hypothenuſe, + CE the Baſe, is to RI, thrice the Radius = 172; ſo is EA, the ſhorteſt Leg to IB = IA = the Angle ACE, required. Or, more briefly, by taking Half the two firſt Terms, it will be,

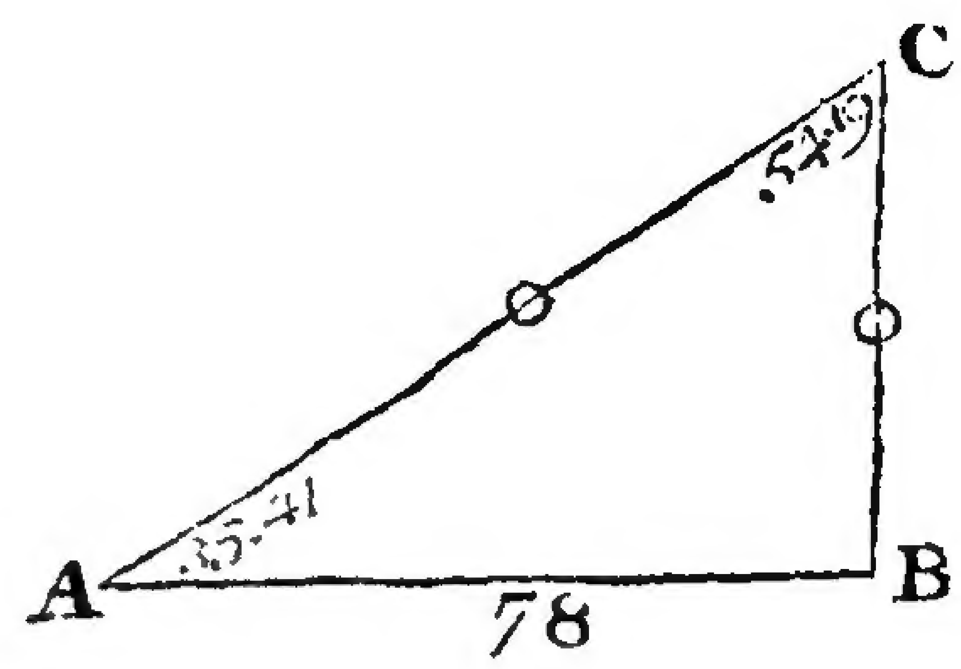
As CA + $\frac{CE}{2}$: to $\frac{RI}{2}$ = 86 : : ſo is AE the ſhorteſt Leg: to its oppoſite Angle ACE required. *Q. E. D.*

NOTE. Theſe four *Axioms* will anſwer all the Caſes of *Right* and *Oblique Angled Triangles*, except the laſt Caſe in Obliques, which will require ſome further Aſſiſtance, and will be ſhewn when we come to treat of that Caſe.

[*] 86 = *Radius and Half* of a Circle whoſe Circumference is 360, or, 85.94368.

CASE I.

The Acute Angles, and one Leg given; to find the Hypothenufe and the other Leg.



Given { Angle A = 35·41, Angle C = 54·19, Bafe AB = 78 } to find { Hypothen. AC and Perpendic. BC.

(1ft.) Find the Natural Radius by Axiom I.

```
      35·7            35·7
      35·7               3
     -----           -----
      2499           107.1
     1785            300.
    1071             -----
   ---------         407.1
   1274·49
         4
   --------
407.1)5097.96(12.5
      4071    (54·3
      -----   -----
      10269   66.8 Natural Radius
       8142
      -----
      21276
      20355
      -----
        921
```

(2d.) Find the Hypothenufe by Axiom I.

```
Angle C  :  Bafe  ::  Nat. Rad.
As 54·3 ---- 78 ------ 66.8
                        78
                      ----
                      5344
                     4676
                    ------
        54·3)5210·4(95·9 + Hypothenufe
             4887
             ----
             3234
             2715
             ----
             5190
             4887
             ----
              303
```

(3d.) Find the Perpendicular by Axiom III.

```
                     96  To Hypothenufe
                     78  Add the Bafe
                    ---
                    174  Sum multiply
                     18  by Difference
                    ---
                   1392
                   174
                   ----
Extract the Root   3132(55·9 + Perpendicular
                   25
                   ---
               105)632
                   525
                   ----
              1109)10700
                    9981
                    ----
                     719
```

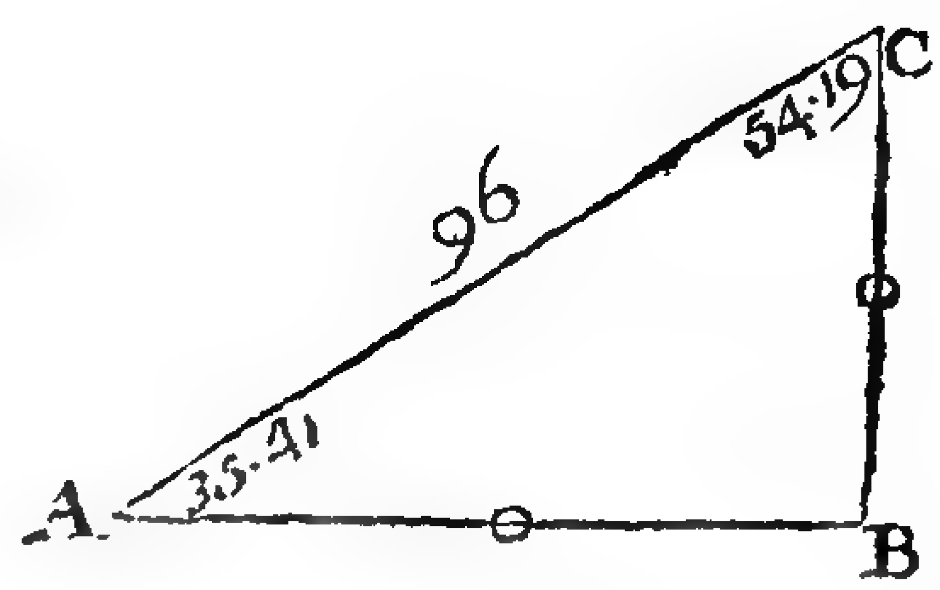
Anfwer, { Hypothenufe, 95·9 +, or 96. Perpendicular, 55·9 +, or 56.

CASE

C A S E II.

The Hypothenuſe and Angles given, to find the Two Legs.

Given { Hypoth. AC = 96, Angle A = 35.41, Angle C = 54.19 } to find { Perpendic. CB and Baſe AB.



(1ſt.) Find the Natural Radius by Axiom I.

```
        35.7
        35.7
        ----
        2499
       1785
      1071
      -------
      1274.49
            3
      --------
(1.000) 3.823.47
        57.3 Add
        ----
        61.1 Natural Radius
```

(2d.) Find the Perpendicular by Axiom I.

```
Nat. Rad. : Hypoth. :: Angle A
As 61.1 —— 96 —— 35.7
                  96
                 ----
                 2142
                3213
                -----
       61.1)3427.2(56 Perpen.
            3055
            ----
             3722
             3666
             ----
               56
```

(3d.) Find the Baſe by Axiom III.

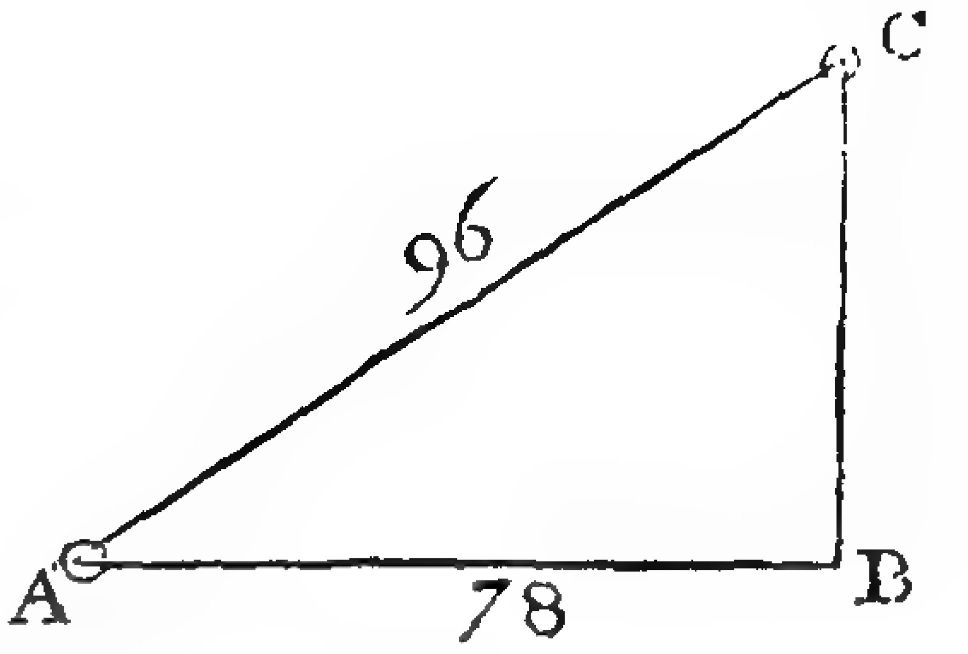
```
                  96 To Hypothenuſe
                  56 Add Perpendicular
                 ---
                 152 Sum multiplied by
                  40 The Difference
                ----
Extract the Root 6080)77.9 + Baſe
                 49
                 --
             147)1180
                 1029
                 ----
            1549)15100
                 13941
                 -----
                  1159
```

Anſwer, { Perpendicular, 56. Baſe, 77.9 +, or 78.

CASE III.

The Hypothenufe and One Leg given, to find the Angles and the other Leg.

Given { Bafe AB = 78, Hypoth. AC = 96 } to find { Perpendic. CB and Angles A & C.



(1ft.) Find the Perpendicular by Axiom III.

96 To Hypothenufe
78 Add the Bafe

174 Sum multiply
18 By Difference

1392
174

Extract the Root 3132(55.9 + Perpendicular
25

105)632
525

1109)10700
9981

719

(2d.) Find the Angle by Axiom IV.

To Hypothenufe 96
Add half longer Leg 39

Sum 135 : Fixed Number 86 : : Perpendicular
56
86

336
448

135)4816.(35.67 + Angle A.
405

766
675

910
810

1000
945

55

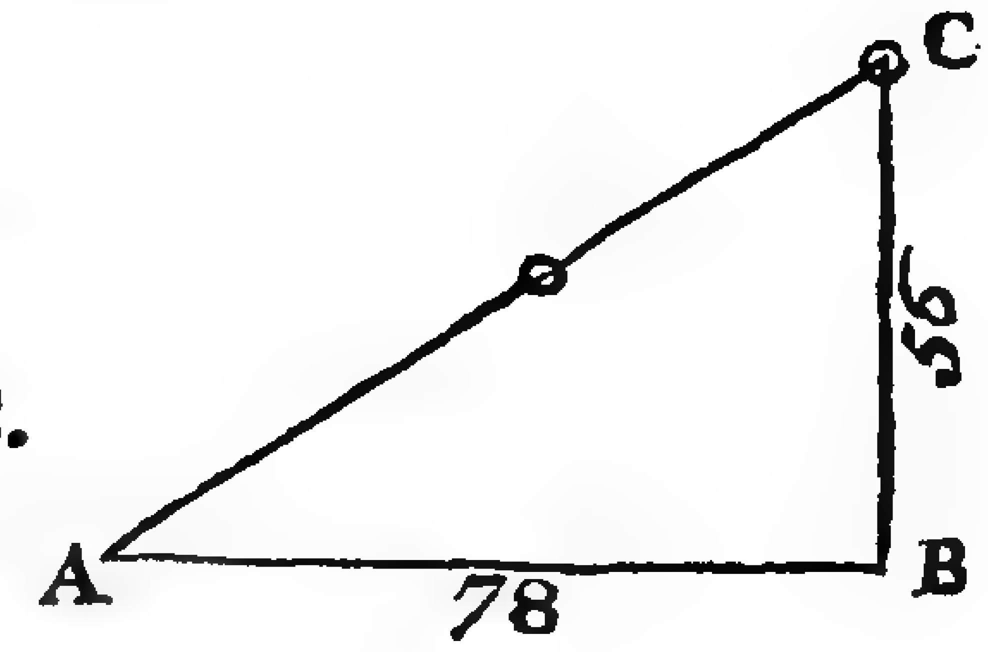
Anfwer, { The Angle, 35° 41′ nearly. The Perpendicular, 55.9 +, or 56.

CASE

CASE IV.

The Two Legs given; to find the Hypothenuſe and the Angles.

Given { Baſe AB = 78, Perpend. CB = 56 } to find { Hypoth. AC. and Angles A & C.



(1ſt.) Find the Hypothenuſe by Axiom II.

```
Perpendicular 56        78 Baſe
   ſquar'd    56        78 ſquar'd
             ----      ----
              336       624
             280       546
             ----      ----
             3136      6084 Square of Baſe
             ----      3136 Square of Perpendic.
                       ----
Extract the Root       9220(96 The Hypothenuſe
                       81
                       ----
                    186(1120
                       1116
                       ----
                          4

To  Hypothenuſe   96
Add half longer Leg 39
                  ----  :  Fixed Number  : :  Perpendicular
          Sum 135 ------      86      ------      56
                                                  86
                                                 ----
                                                  336
                                                 448
                                                 ----
                                          135)4816.(35.674 + Angle A
                                                 405
                                                 ----
                                                  766
                                                  675
                                                  ----
                                                   910
                                                   810
                                                   ----
                                                   1000
                                                    945
                                                    ----
                                                     550
                                                     540
                                                     ----
                                                      10
```

Anſwer, { The Hypothenuſe, 96.
The Angle, 35°.674, or 35° 41′ nearly.

D NOTE.

NOTE. Thus all the Cafes of *Right Angled Triangles*, are eafily and readily anfwered: and by the fame Rules, and with the like Eafe may the *Oblique Angled Triangles* be anfwered, as will evidently appear in the following Cafes.

Of Oblique TRIANGLES.

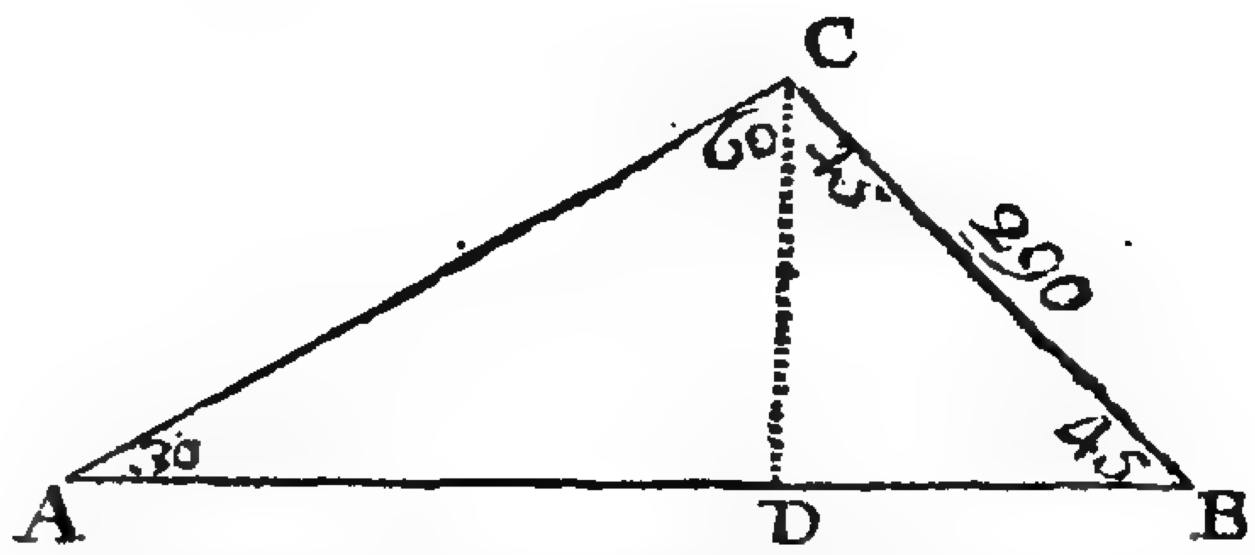
IN the Solution of an *Oblique Triangle*, it is neceffary, by this Method, to divide it into Two *Right Angled Triangles*, by means of a *Perpendicular*, which muft always fall from the End of a given Side, and oppofite to a given Angle.

By this means the *Perpendicular* will fometimes fall *within*, and fometimes *without* the Triangle: When it falls within, it falls upon fome Part of the Bafe, or longeft Side; but when it falls without, it falls upon one of the fhorter Sides continued. In either Cafe, there are Two *Right Angled Triangles* made, and the Angles, or Sides fought, are found as if they were Parts of *a Right Angled Triangle*, by the foregoing Axioms; but it requires *Two* or *Three* Operations.

CASE

CASE I.

Two Angles, and a Side oppofite to one of them, given; to find the other two Sides.



Given { Angle A 30, Angle B 45, Side BC 290 } to find { Side AC and Side AB.

```
  45        45
  45         3
 ----      ----
  225       135
 180        300
 ----      ----
 2025       435
    4
 ----
435)8100(18.6
    435  45.
    ----
    3750 63.6 Natural Radius
    3480
    ----
     2700
     2610
     ----
       90
```

```
   30
   30
  ---
  900
    3
  ---
 2700
 57.3
Natural Radius 60.0
```

(1ft.) Find the Perpendicular CD.

```
     N. Rad.      Hypoth.      ∠ at B
As 63.6  ———————  290  ———————  45
                   45
                 ----
                 1450
                 1160
                 -----
        63.6)13050(205 Perpendicular CD.
             1272
             ----
              3300
              3180
              ----
               120
```

(2d.) Find the Side AC.

```
∠ at A      Perp. CD      N. Rad.
As 30  ———   205   ———    60
              60
             ----
  30)12300(410 The Side AC Hypothenufe.
     120
     ---
      30
      30
      --
       0
```

(3d.) Find the Side AD.

```
       To Hypoth. AC  410
     Add Perpend. CD  205
                      ---
                 Sum  615
Multiply by Difference 205
                      ----
                      3075
                     12300
                     ------
Extract the Root    126075(355 AD
                     9
                    ---
                  65)360
                     325
                     ----
                 705)3575
                     3525
                     ----
                       50
```

(4th.) Find the Side DB.

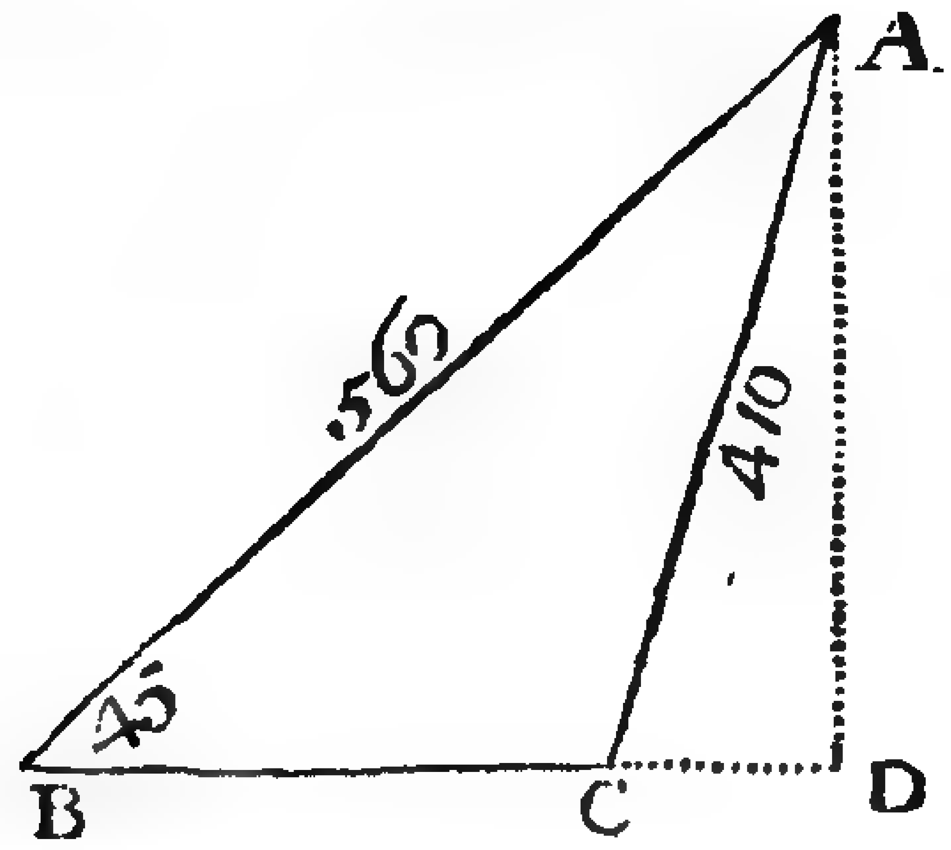
```
       To Hypoth. CB  290
     Add Perpend. CD  205
                      ---
                 Sum  495
Multiply by Difference  85
                      ----
                      2475
                     3960
                     -----
Extract the Root    42075(205 DB.
                    4
                   ---
               405)2075
                   2025
                   ----
                     50
```

Anfwer, { The Side AC 410. The Side AB 560. The Angle C 105.

CASE

CASE II.

Two Sides, and an Angle opposite to One of them, being given; to find the rest.



Given			to find	
	Side AB	560		Angle C
	Side AC	410		and
	Angle B	45°		Side BC.

(1st.) Find the Side AD.

Nat. Rad. —— Hyp. BA —— ∠B
As 63.6 —— 560 —— 45

560
45
2800
2240
63.6,25200.0(396 AD
1908
6120
5724
3960
3816
144

(2d.) Find CD.

To Hypoth. CA 410
Add Perpend. AD 396
Sum 806
Multiply by Difference 14
3224
806
Extract the Root 11284(106 CD
1
206(1284
1236
48

(3d.) Find the Angle DAC.

To Hyp. CA 410
Add ½ Perp. AD 198

Fix'd Num. —— Side CD
As 608 —— 86 —— 106

106
86
636
848
608)9116(14.9+, or 15° ∠ A
608
3036
2432
6040
5472
568

(4th.) Find Side BD.

To Hypoth. BA 560
Add Perpend. DA 396
Sum 956
Multiply by Differ. 164
3824
5736
956
Extract the Root 156784(395.9 + or 396 BD
9..
69 667
621
785(4684
3925
7909)75900
71181
4719

Then from BD = 396
Take —— CD = 106
Remains — BC = 290 Required.

Hence we find by Inspection.
The Angle ACD = 75
The Angle ACB = 105
and The Angle BAC = 30° Requ.

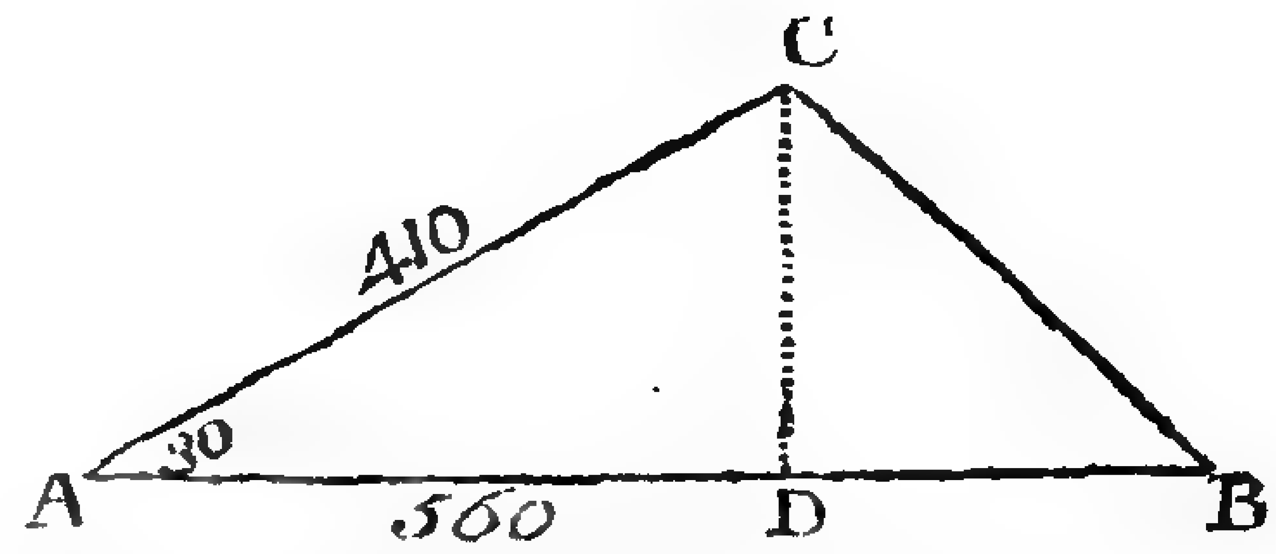
☞ In this, and several of the following *Cases* and *Problems*, the Operation for finding the *Natural Radius* is omitted, for Want of Room.

CASE

CASE III.

Two Sides, with the Angle comprehended by them, given; to find the rest.

Given { Side AC 410, Side AB 560, Angle A 30° } to find { Side BC and Angles B and C.



(1st.) Find the Perpendicular CD.

```
    N. Rad.   Hypoth. AC   ∠ at A
As   60  ———   410  ———   30
                30
             ————
     60)12300(205 Perpendicular CD
        120
        ———
          300
          300
          ———
            0
```

(2d.) Find the Part AD.

```
  To Hypoth. AC 410
 Add Perpend. CD 205
                 ———
             Sum 615
Multiply by Differ. 205
                    ———
                   3075
                  12300
                  —————
Extract the Root 126075(355 Base AD
                 9....
                 ————
              65)360
                 325
                 ———
             705)3575
                 3525
                 ————
                   50

     From AB = 560
     Take AD = 355
               ———
  Remains BD = 205
```

(3d.) Find the Side BC.

```
BD squar'd 205          205 CD squar'd
           205          205
           ————         ————
           1025         1025
           4100         4100
           —————        —————
To Square of DB 42025   42025
Add Square of CD 42025
                 —————
Extract the Root 84050(289.9 +, or 290 BC
                 4
                 ——
             48)440
                384
                ———
            569)5650
                5121
                ————
           5789)52900
                52101
                —————
                  799
```

(4th.) Find the Angle B.

```
To 290    the Side BC
Add 102.5 half DB
——————    Fix'd Num.   Side DC
As 392.5 ——  86  ——   205
                       86
                      ————
                      1230
                      1640
                      —————
        392.5)17630.0(44.9 +, or 45° ∠ B
              15700
              —————
              19300
              15700
              —————
              36000
              35325
              —————
                675
```

The Angle A = 30, added to Angle B = 45, and then subtracted from 180, leaves 105 for Angle C.

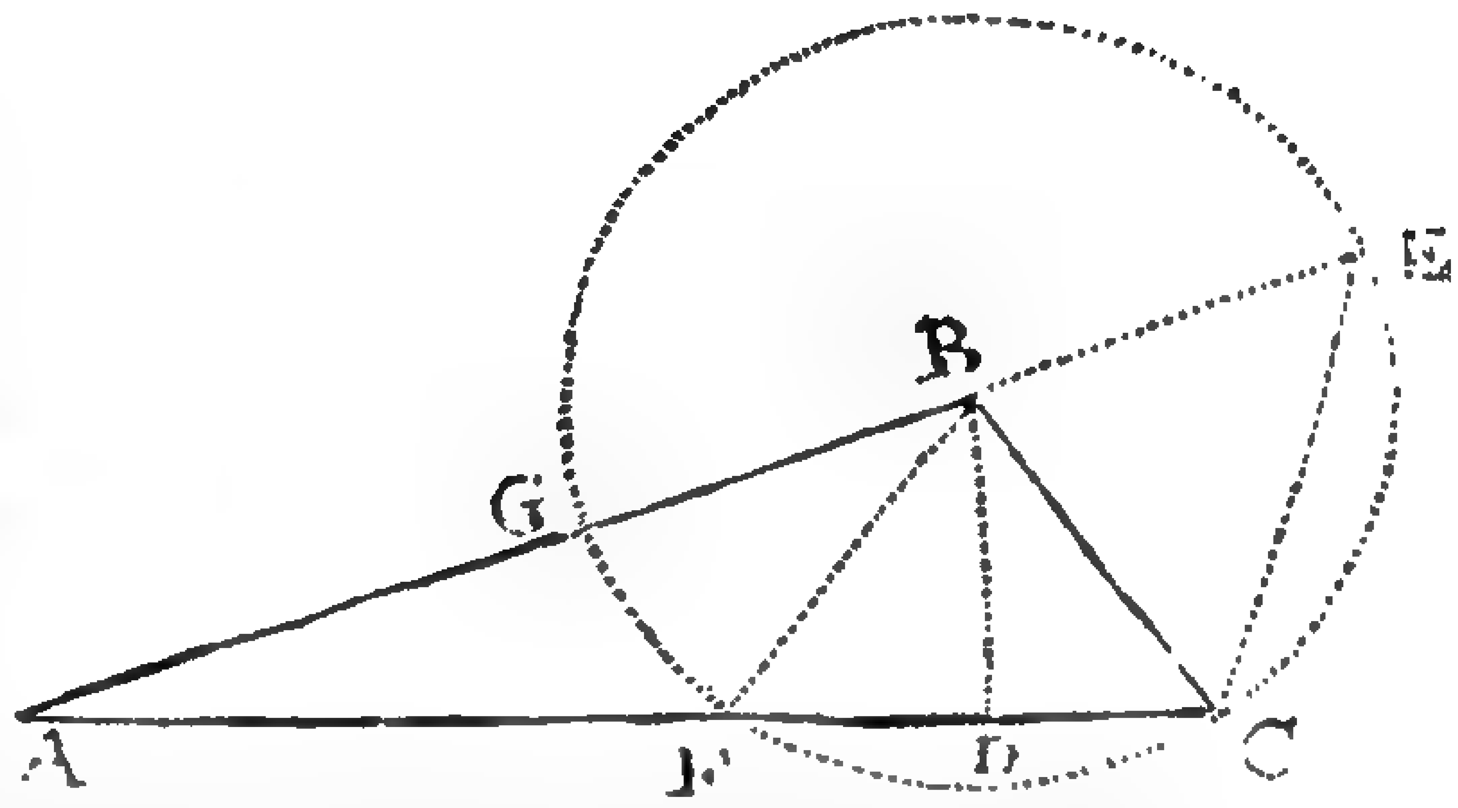
Answer, { The Side BC 290, The Angle B 45°, The Angle C 105°.

CASE IV.

The Three Sides given; to find the three Angles.

In all Triangles, as the Baſe or greater Side is to the Sum of the other Two Sides; ſo is the Difference of the Sides to the Difference of the Segments of the Baſe; which Half Difference, added to Half the Baſe, the Sum will be the *Greater Segment*, upon which the Perpendicular falls: But if ſubtracted from Half the Baſe, the Remainder will be the *Leſs Segment:* So will the *Oblique Triangle* be reduced to two Right Angled ones, and may be anſwered after the ſame Manner as before.

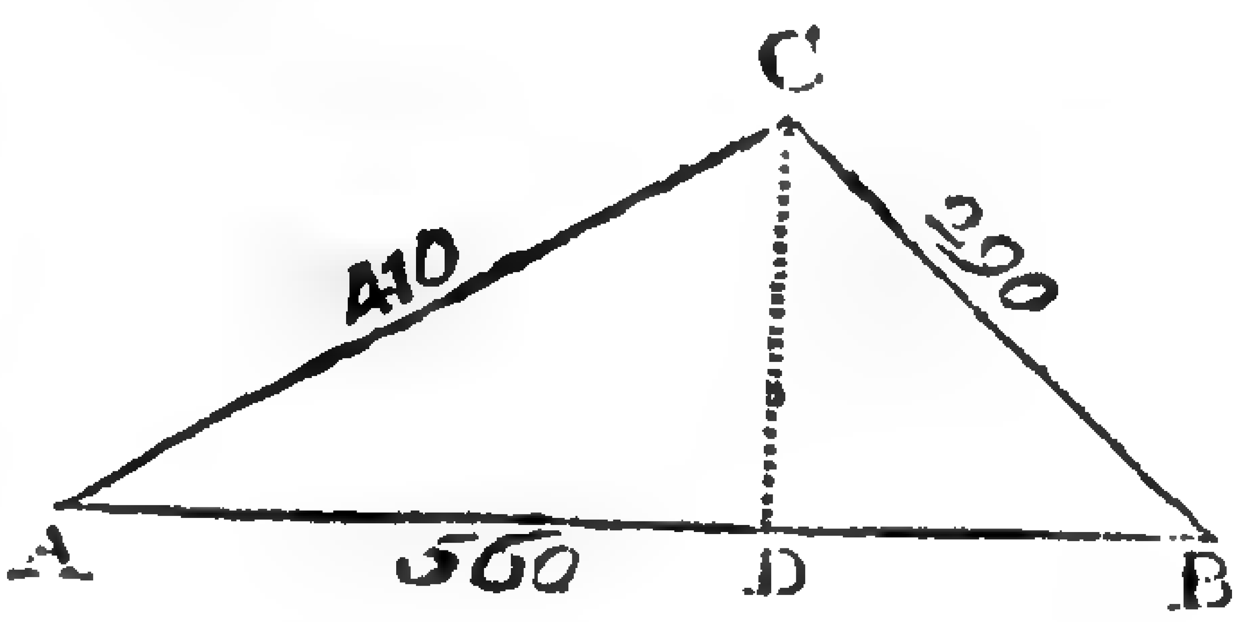
CONSTRUCTION.



Let ABC be the Triangle propoſed, whoſe three Sides AB, AC, and BC are given. On B, as a Centre, with the Radius BC, deſcribe the Circle GFCEB; continue the Line AB till it cuts the Circle in E; let fall the Perpendicular BD; draw the Lines EC and BF. Then will AE be the Sum of the two Sides AB and BC, and AG, their Difference. Alſo AC = AD + DC, the Sum of the Segments of the Baſe, and AF = AD — FD, their Difference.

DEMONSTRATION. Becauſe the Triangles ABF, and AEC, are ſimilar, it will be, as AC : AE : : AG : AF. Q. E. D.

Given { Side AB 560, Side AC 410, Side BC 290 } to find { Angle A, Angle B, Angle C.



(1ſt.) Find the Segments of the Baſe.

As 560 —— 700 —— 120

700
120
14000
700
560)84000(150 Difference
560
2800
2800
0

To ½ Baſe 280
Add ½ Diff. 75
G. Seg. AD 355

From ½ Baſe 280
Subtract ½ Diff. 75
Leſſer Seg. DB 205

(2d.) Find the Perpendicular CD, by Caſe III. = 205.

(3d.) Find the Angle at A.

To 410 Side AC
Add 177 half AD
As 587 —— Fix'd Num. 86 —— Side DC 205 —— 30 ∠ A

(4th.) Find the Angle at B.

To 290 Side BC
Add 102 half DB
As 392 —— Fix'd Num. 86 —— Side DC 205 — 49.9+, or 45 ∠ B

Then the Angle A = 30°; added to B = 45°, and ſubtracted from 180°, leaves 105° for the Angle C, which were the Angles required.

THE

THE USE OF TRIGONOMETRY

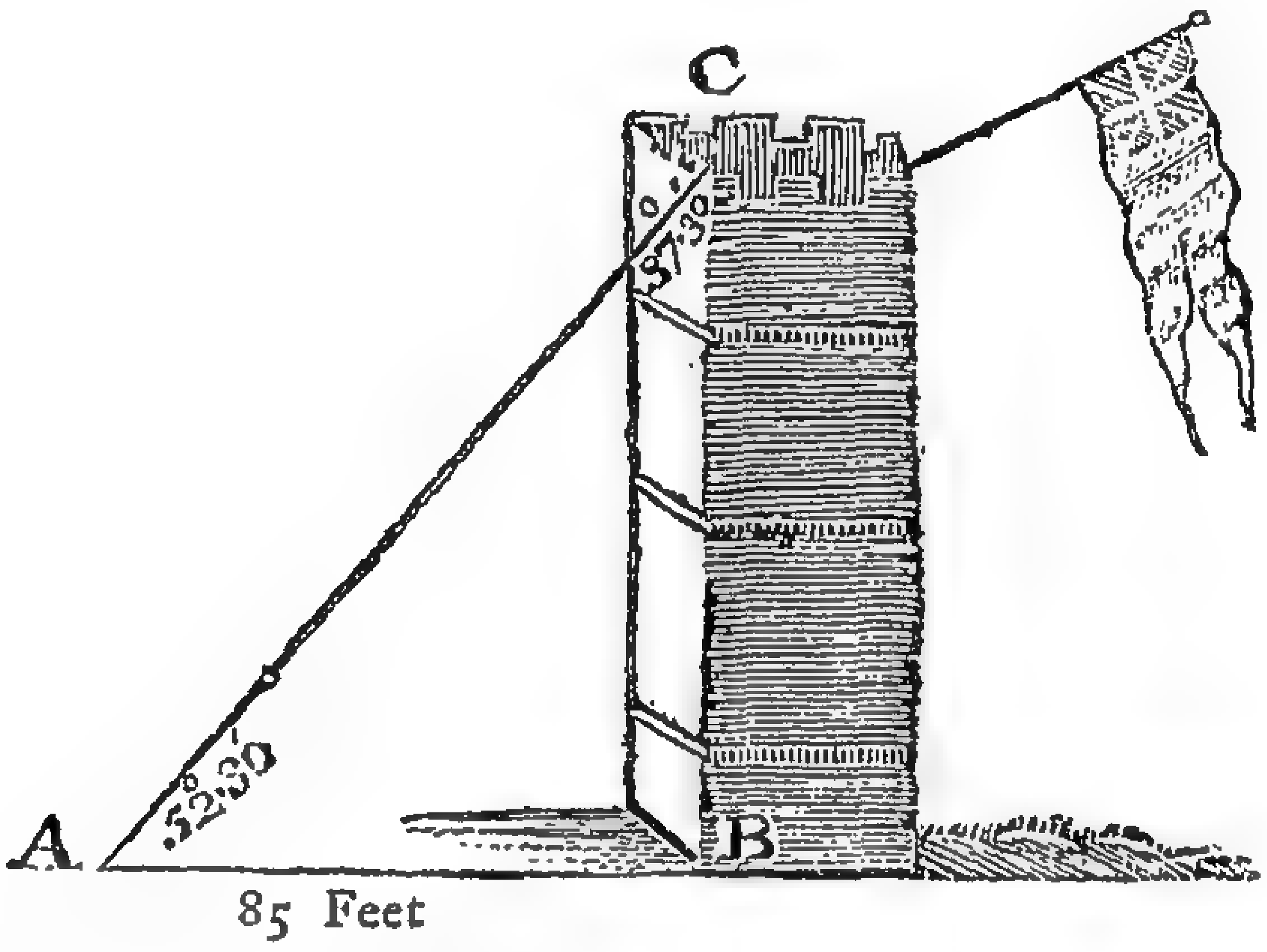
Exhibited in the Solutions *of a Number of interesting* Problems; *many of which every Day occur; are of the greatest Utility in the* Army, Navy, *&c. and cannot be answer'd without it.*

PROBLEM I.

To take the Height of any *accessible Object* at one Station.

First, with a *Quadrant*, by looking through the Sights to the Top of the Tower, find the Quantity of the Angle A, which suppose 52° 30'; then measure the Distance AB, which suppose to be 85 Feet; from hence you may proceed to find the *Height*, by Case I. of Right Angled Triangles.

37.5
37.5
————
1875
2625
1125
————
1406.25
3
————
1000)4.218.75
57.3
————
61.51875 Natural Radius
————
But 61.52 is exact enough in Practice.



(1st.) Find the Hypothenuse AC.

Angle C : Base : : Nat. Rad.
As 37.5 — 85 —— 61.52
85
————
30760
49216
————
37.5)5229.20.(139.44+Hyp.
375
————
1479
1125
————
3542
3375
————
1670
1500
————
1700
1500
————
200

(2d.) Find the Perpendicular BC.

To Hypothenuse 139.44
Add Base 85.
————
Sum 224.44
Multiply by Difference 54.44
————
89776
89776
89776
112220
————
Extract the Root 12218.5136(110.537 Perpend.
1 . . . .
————
21)22
21
————
2205)11851
11025
————
22103)82636
66309
————
221067)1632700
1547469
————
85231

Answer, 110.537 +, or 110 Feet, and above ½: The Height required.

Note; That in this, and all such Cases, you must add the *Height* of your *Eye*, or *Instrument*, to the *Altitude* before found.

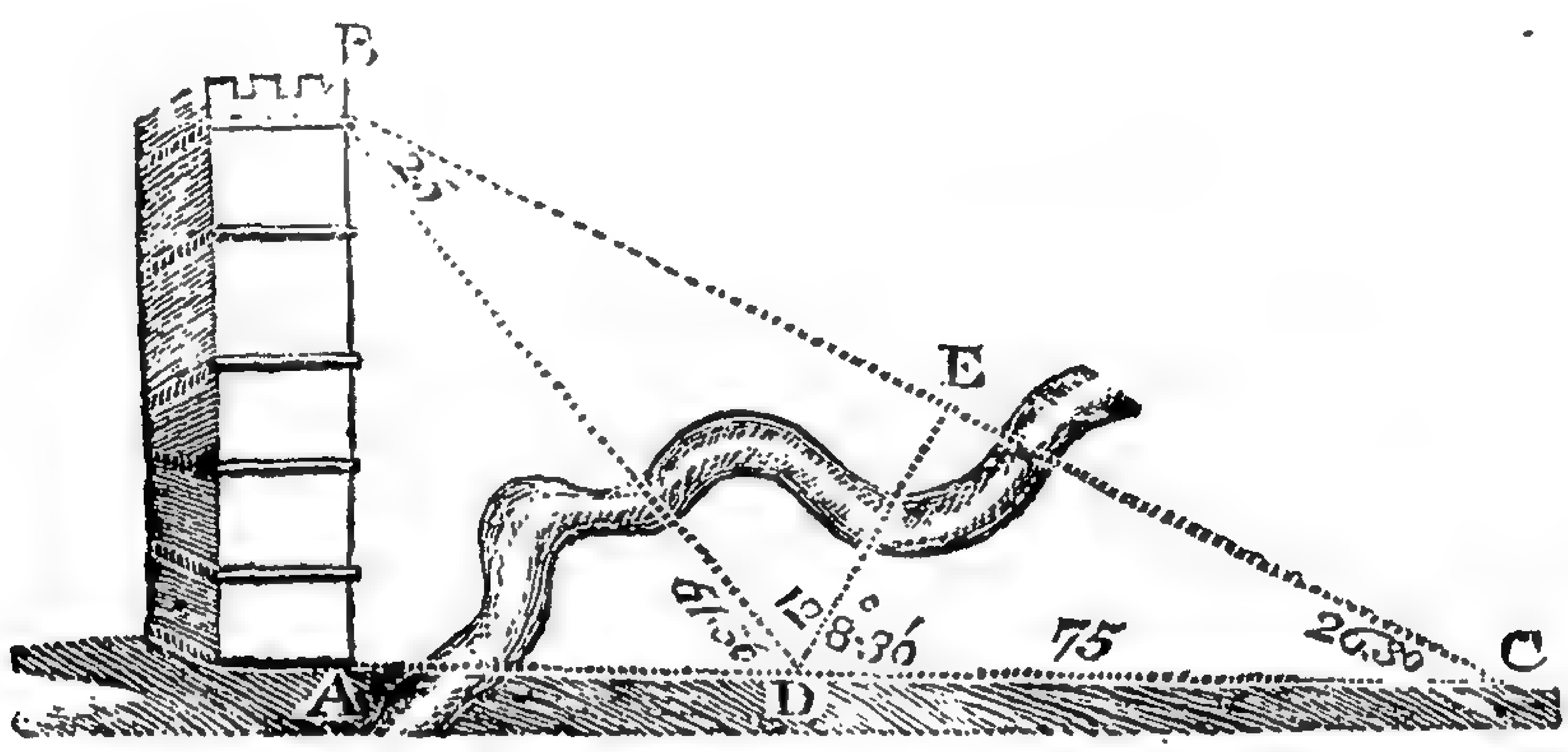
PROBLEM

PROBLEM II.

To meaſure an *Inacceſſible Altitude.*

Let AB, in the following Figure, be a *Church, Tower,* or *Fort,* whoſe Height is required; but by Reaſon of a River, or ſome other *Obſtacle,* it is *inacceſſible*; that is, you cannot come to the Foot of it, by Reaſon of the *Water,* &c.

Firſt, with a *Quadrant,* take the Angle of *Altitude* at C, which ſuppoſe 26° 30′. Then meaſure in a Right Line towards the *Tower* to D, any Diſtance, ſuppoſe 75 Feet, and at D obſerve again the Angle of *Altitude,* which let be 51° 30′.



Then; the two Viſual Lines CB and DB, with the Diſtance DC, make the Oblique Triangle CBD, in which are given—All the Angles and Side CD. The Angles BCD being 26° 30′, and the Complement of ADB 51° 30′ to 180, is the Obtuſe Angle BDC 128° 30. Conſequently, the third Angle CBD, at the Top, is = 25°.

(1ſt.) Find the Perpendicular DE in Triangle DBC.

Nat. Rad. : Op. Side DC : : Ang. C : Perp. DE
As 59.4 —— 75 —— 26.5 — 33.46

(2d.) Find the Viſual Line BD in Triangle BDE.

Ang. B : Op. Side DE : : N. Rad. : Side BD
As 25 —— 33.46 — 59.17 — 79.19

(3d.) Find the Height AB in the Right Angled Triangle ABD.

Nat. Rad. : Op. Side BD : : Ang D : Height
As 65.7 —— 79.19 —— 51.5 — 62 AB

(4th.) Find the Diſtance AD in the Triangle ABD.

N. Rad. : Op. Side BD : : Ang ABD : Dif. AD
As 61.7 — 79.19 —— 38.5 —— 49.41

Anſwer, { 62 Feet the Height.
49.41, or 49½ Feet the Diſtance from the ſecond Station.

NOTE: The Line BD is the Length of a *Scaling Ladder,* which would reach from the *Station* at D over the *Foſs* or *Ditch,* to the *Top* of the *Tower* at B.

PROBLEM

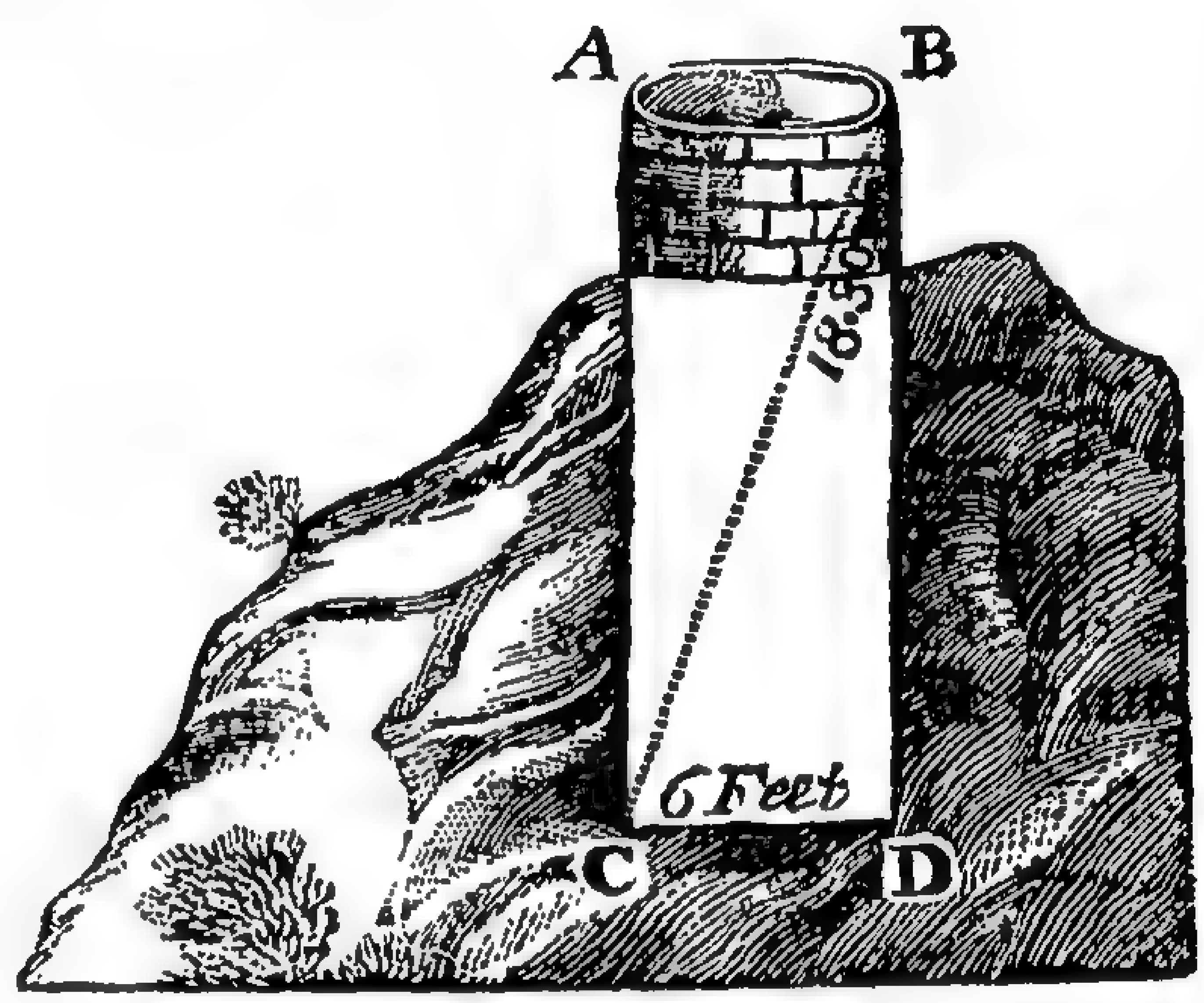
PROBLEM III.

To meaſure the *Depth* of a *Well*, or the *Height* of an *Object* from the *Top* of it.

Firſt, look through the Sights of the *Quadrant* to the Bottom of the oppoſite Side the Well at C, ſo you will have the *Angle* CBD; next, take the *Breadth* AB at the Top, which is equal to CD at the Bottom: Then, by Caſe I. of Right Angle Triangles, you may eaſily find the *Depth* BD required.

Suppoſe the Angle at B, by Obſervation, to be 18° 30′, and the Breadth at the Top 6 Feet: What's the *Depth?*

18.5
18.5
———
925
1480
185
———
342.25
3
———
1.026.75
57.3
———
Natural Radius 58.32675 but 58.3 is enough.



(1ſt.) Find the Hypothenuſe BC.

∠B Op. Side N. Rad.
As 18.5 — 6 — 58.3
6
———
18.5)349.8.(18.9 + the Line BC
185
———
1648
1480
———
1680
1665
———
15

(2d.) Find the Depth BD.

To Hypoth. BC 18.9
Add Side CD 6.
———
Sum 24 9
Multiply by Differ. 12.9
———
2241
498
249
———
Extract the Root 321.21(17.89 the Depth AB
1
———
27)221
189
———
348)3221
2784
———
3569)43700
32121
———
11579

Anſwer, $\left\{ \right.$ 17.89 +, or 18 Feet, the Depth required.

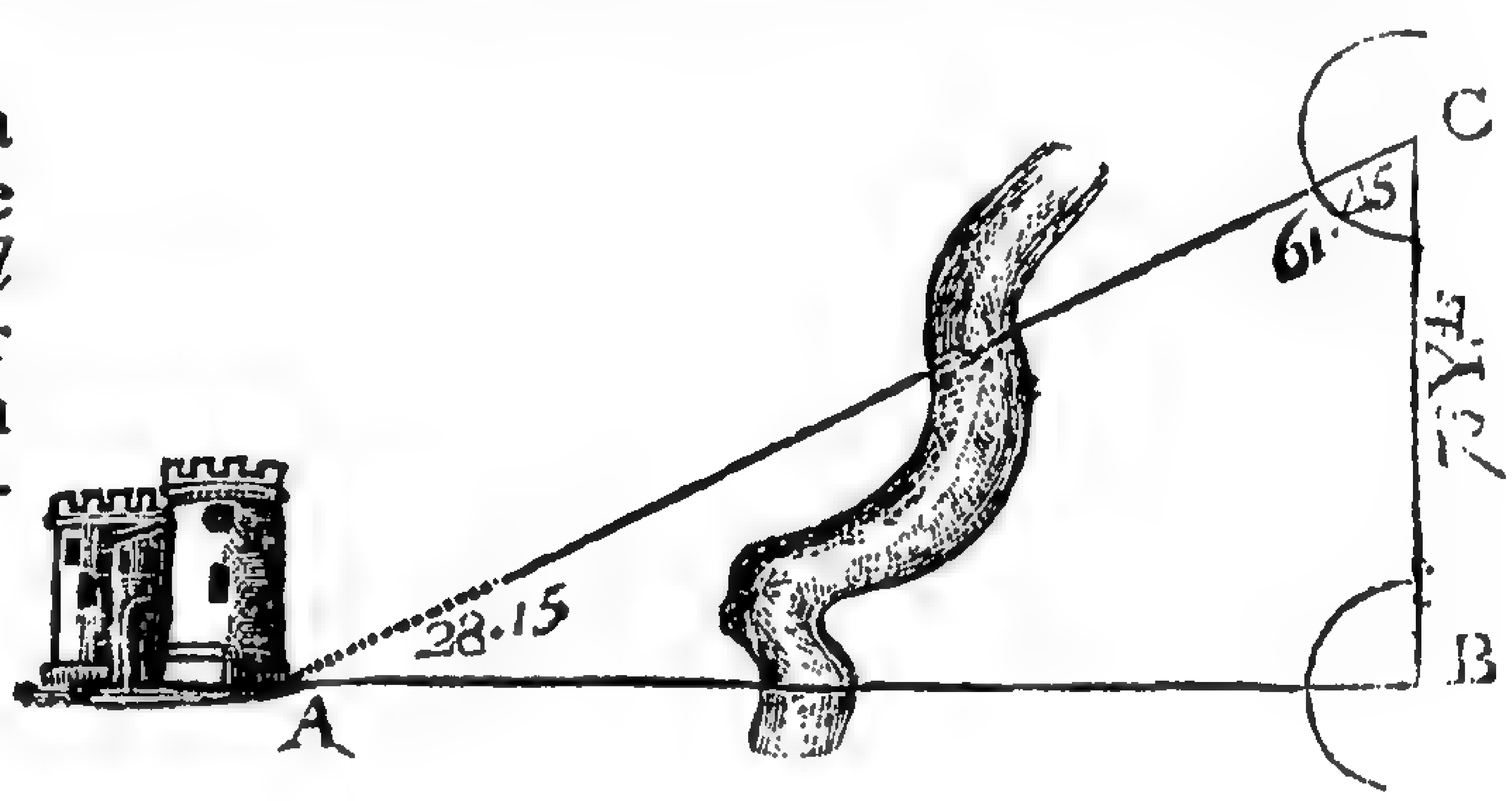
PROBLEM IV.

To meaſure the *Diſtance* of any *Object*.

Suppoſe yourſelf ſtanding at B, and a great Way off, as at A, you ſee a *Fort* or *Caſtle*, &c. or any other *Object*, whoſe *Diſtance* you would find from the Place where you now ſtand.

Firſt, a *Theodolite*, or *Semicircle*, being placed at B, lay the *Index*, with its Sights, on the *Diameter*, where the Degrees begin, and through them view the *Caſtle*, &c. at A. The Inſtrument remaining fix'd in this Poſition, move the *Index* to 90 Degrees, (that being a Right Angle) and view ſome Mark at a Diſtance, (the farther off the better) as at C.—Next meaſuring the Diſtance from B to C, which ſuppoſe 73 Yards, remove your Inſtrument, and ſet it up at C. Then, with the Index laid upon the Beginning of the Degrees, as before, turn the Inſtrument about, till you can ſee your *firſt Station* at B, where faſten it; then turn the Index till you can ſee the Object A, and obſerve what Degrees are cut, as ſuppoſe 61° 45′, which is the Quantity of the Angle where you ſtand; whoſe Complement to 90° is the Angle A.

Now, here are given all the Angles, and one Side of a *Right Angled* Triangle, to find either of the other Sides, which will be the *Diſtance* required.



28.25
28.25
———
14125
5650
22600
5650
———
798.0625
3
———
2 394.1875
57.3
———
59.69 or 59.7 Natural Radius

(1ſt.) Find the Diſtance from C.

Ang. A : Op. Side BC : : N. Rad. : Diſt. CA
As 28.25 — 73 ——— 59.7 — 154.2

Anſ. yards { 135.8 +, or 136 Diſt. from B.
154.2, or 154¼ Diſt. from C.

(2d.) Find the Diſtance from B.

To Hypoth. AC 154.2
Add Side BC 73
———
Sum 227.2
Multiply by Differ. 81.2
———
4544
2272
18176
———
Extract the Root 18448.64(135.8 +, Diſt. from B
1
———
23)84
69
———
265)1548
1325
———
2708)22364
21664
———
700

PROBLEM V.

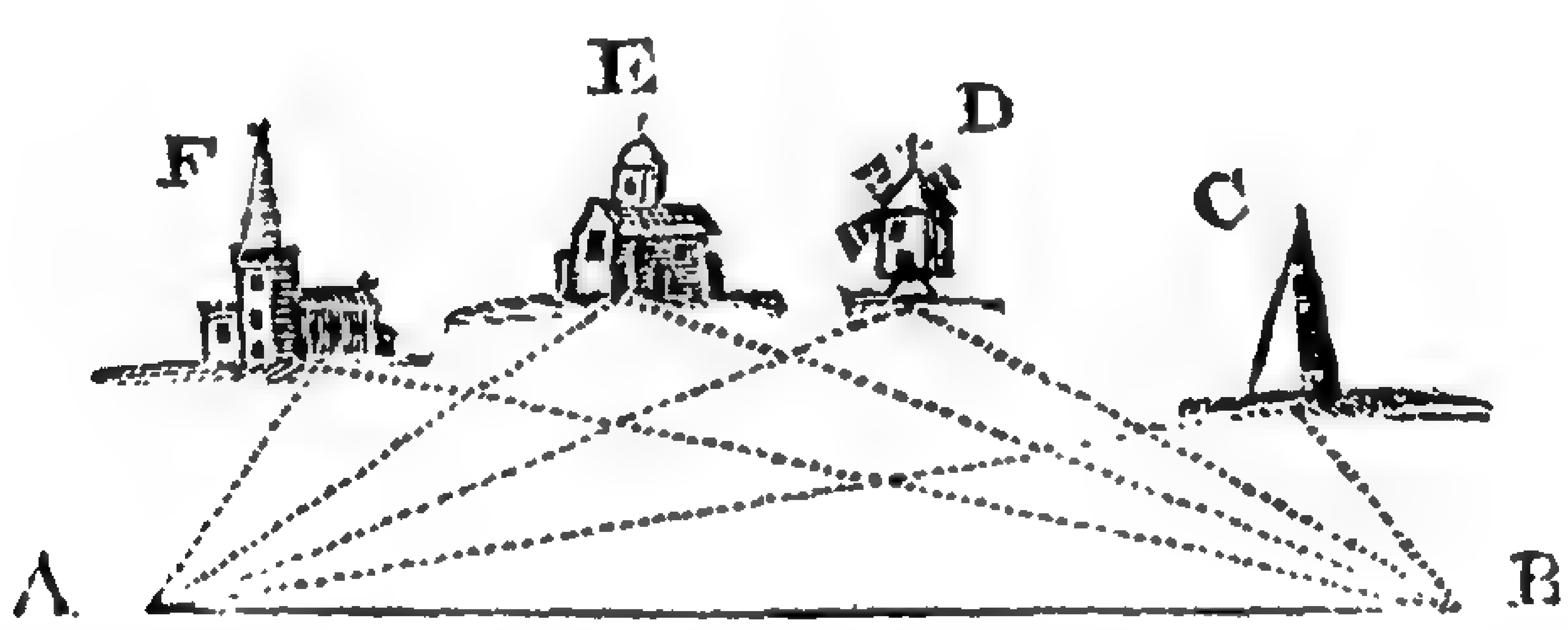
To take the *Diſtances* of ſeveral *inacceſſible Objects*, as *Forts*, *Churches*, in a *Town*, or *Squadron of Ships* at *Sea*, and to delineate them upon Paper.

Firſt, make choice of two places, from either of which you may conveniently ſee all the *Objects*; which two Places let be A and B in the following Figure.—This being done, ſet up your Inſtrument at A, laying the *Index* on the *Diameter*, and turn the whole Inſtrument about, till, through the Sights, you ſee your ſecond Station B. Then, fixing the Inſtrument, direct your Sights to the ſeveral *Objects*, C, D, E, and F; noting down the Degrees cut at each Obſervation, which ſuppoſe to be as in the Table.

Then, remove the Inſtrument to B, laying the *Index* on the *Diameter*, and turn it about, till, through the Sights, you ſee your former *Station* at A; then direct your Sights to every one of the Objects at C, D, &c. ſetting down the Degrees at each Obſervation, as in the Table. Alſo meaſure the *Stationary Diſtance*, and ſet that down.

	C	D	E	F
1ſt Station	11	23	36	59
2d Station	51	31	22	13
Stationary Diſtance 150 Yards.				

Firſt, upon a Piece of Paper draw the Line AB; and from a *Scale of equal Parts*, take off, with your Dividers, the *Stationary Diſtance* = 150, and ſet it from A to B, ſo will A repreſent your *firſt Station*, and B the *ſecond*. Then lay the *Center* of the *Protractor* upon the



Point A, with its Diameter upon the Line AB; keeping it faſt, make Marks by the Edge at 11, 23, 36, 59, and draw Lines from the Point A through each of thoſe Marks. Then upon B place the *Center* of your *Protractor*, its Diameter lying upon the Line AB; make Marks by the Side at 13, 22, 31, 51. Then draw Lines from the Point B through each of theſe Marks, and where the Lines cut the former *correſpondent Lines* there will be found the Places repreſenting theſe *Objects*. Then any of theſe Lines being taken in a Pair of Dividers, and applied to the Scale you laid your *Stationary Diſtance* down by, will give you their *Diſtances*, either from your *Stations* or from one another.

The *Diſtance* of any of theſe *Objects* from *either Station*, &c. may be found by *Calculation*; *one Side* and the *Angles* being given: But I ſhall omit that, on purpoſe to *exerciſe* the *Learner's Genius*, and proceed.

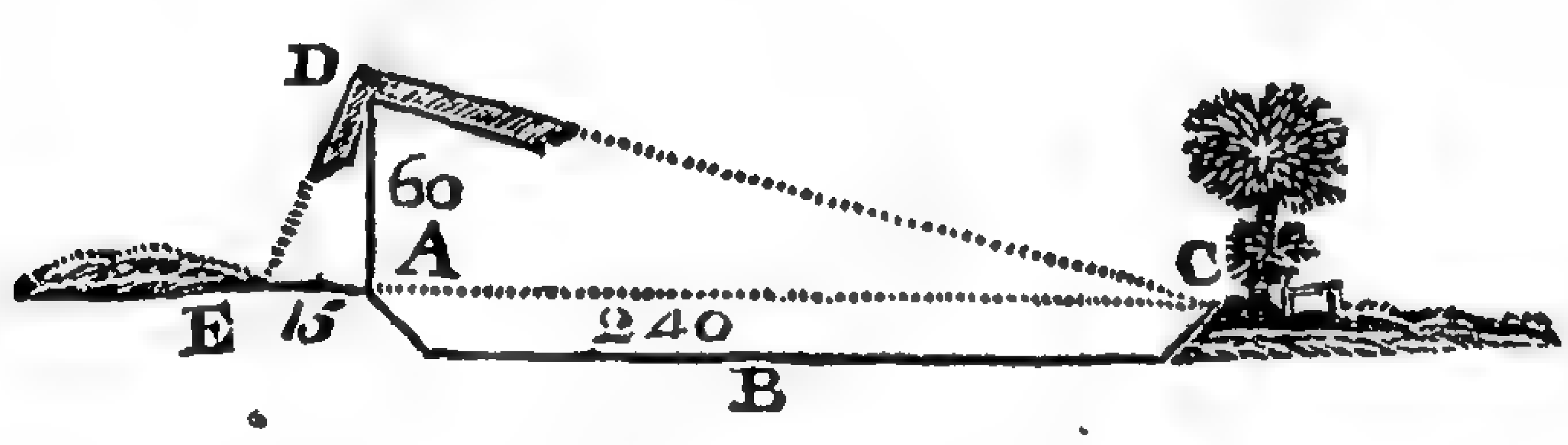
PROBLEM

PROBLEM VI.

To take the *Diſtance*, upon level Ground, of any *inacceſſible Tree, Fort*, &c. or *Breadth* of a *River*, by a *Common Square*.

Suppoſe there is a River, as ABC, whoſe *Breadth* you want to know. —Firſt, upon the Bank, at A, ſet up a Stick, AD, which ſuppoſe to be 5 Feet, or 60 Inches high; then fixing your *Square* on the Top, at D, look by the Side of it till you ſee the Edge of the oppoſite Shore C, and faſten it, as it may not go from that Poſition. This done, extend a Thread from D, by the other Side of the *Square* till it touch the Ground at E. Then meaſure the Diſtance EA, which ſuppoſe 15 Inches, (or 1 Foot 3 Inches) and you may find AC (by Reaſon of ſimilar Triangles) thus.

Diſt. EA : Side DA : : Side DA : Diſt. AC
As 15 —— 60 —— 60 —— 240 Inches,
which, reduc'd to Feet, give 20 for the Breadth of the *River* ſought.



Note. There are various Ways of taking *Heights* and *Diſtances*; but the *beſt* is to take the Angles for *Heights* by a *Quadrant*; and the Angles for *Diſtances* by a *Semicircle* or *Theodolite*; and calculate by the foregoing *Axioms*. In all *Heights* the *Triangle* ſtands *upright*; but in *Diſtances*, it is ſuppoſed to lie *flat* or *horizontal*.

☞ I ſhall here ſhew the Learner, how to take the *Breadth* of a *River*, or a *ſmall Diſtance*, without any *Inſtrument whatever*; which is thus. Standing upon the Bank, bring down the Edge of your *Hat*, till it appears to touch the oppoſite Side, then ſteady your Head by laying your Hand under your Chin, and turn yourſelf towards ſome *level Ground*, obſerving where the Edge of your *Hat* glances upon it; for then, the *Diſtance* from you to that *Place*, is *equal* to the *Breadth* of the *River*, or *Diſtance* required.

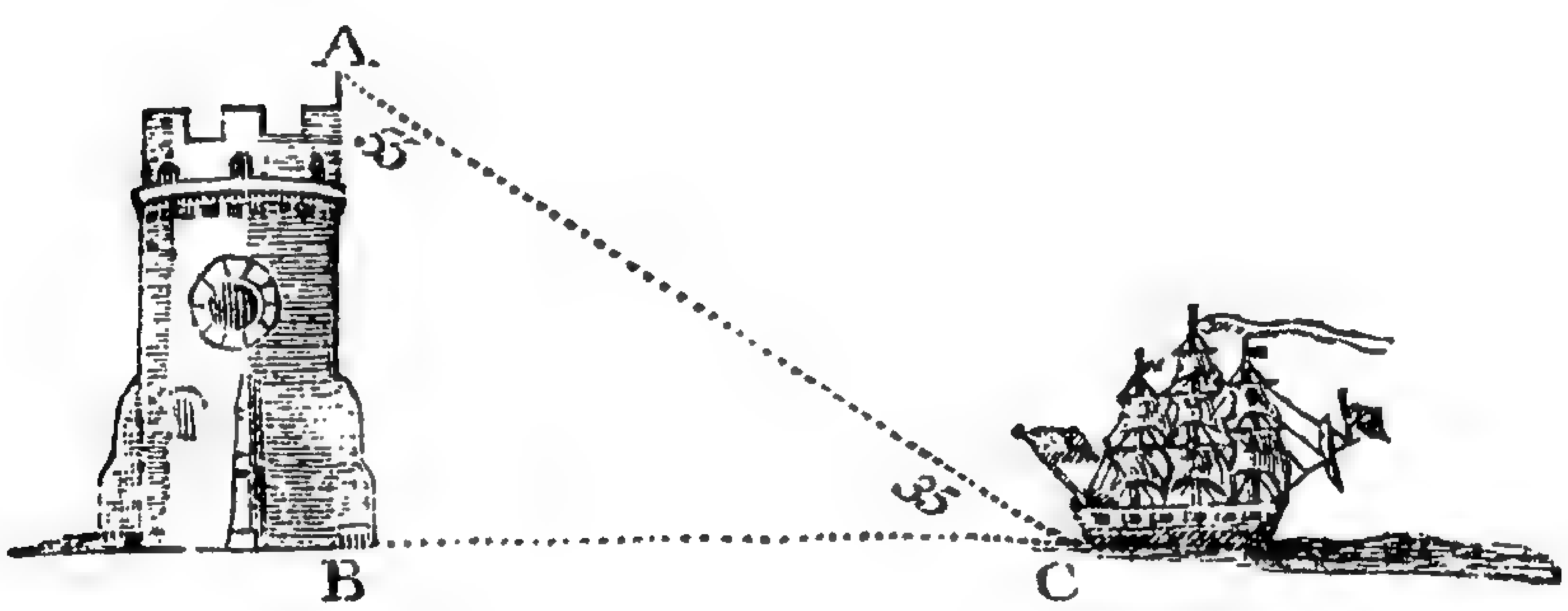
PROBLEM

PROBLEM VII.

To find, from the Top of a *Fort*, or *Tower*, how far any *Tree*, *Ship*, &c. is from you.

Let A be the Top of a *Tower* or *Caſtle* ſtanding by the *Sea Side*; and let C be a *Ship* at Sea, or lying at Anchor, and you would know how far that Ship is off the *Caſtle Wall*.

With your *Quadrant* or *Semicircle*, direct your Sights from the Top of the *Tower* to the Place where the *Ship* is, and take the Angle, which we will ſuppoſe to be 55 Degrees. Then the *Caſtle Wall* being known before to be 143 Feet high, you may eaſily find the *Diſtance* of the *Ship* from the *Wall* in this Manner.



(1ſt.) Find the Side AC.

Ang C	: Height Wall	: : Nat. Rad.	: AC
As 35	—— 143	—— 61	— 249.2

(2d.) Find the Baſe BC.

N. Rad.	: AC	: : Ang. A	: Diſt. BC
As 67	—— 249.2	—— 55	—— 204.56

By this Method you may eaſily diſcover if a *Fleet* of *Ships*, or one *ſingle Ship*, at Sea, makes towards you or not. For having obſerved from the Top of the *Fort* the Angle from thence to the *Ship*, and noted it down, reſt a little Time, and obſerve again: Then, if the Angle be *bigger* than before, the Ship is *departing* from you; but if *leſs* ſhe is *making towards* you.

PROBLEM VIII.

To take the *Perpendicular Height* of a *Hill* or *Mountain*, and alſo the *Horizontal Line*, or *Baſe*, on which it ſtands.

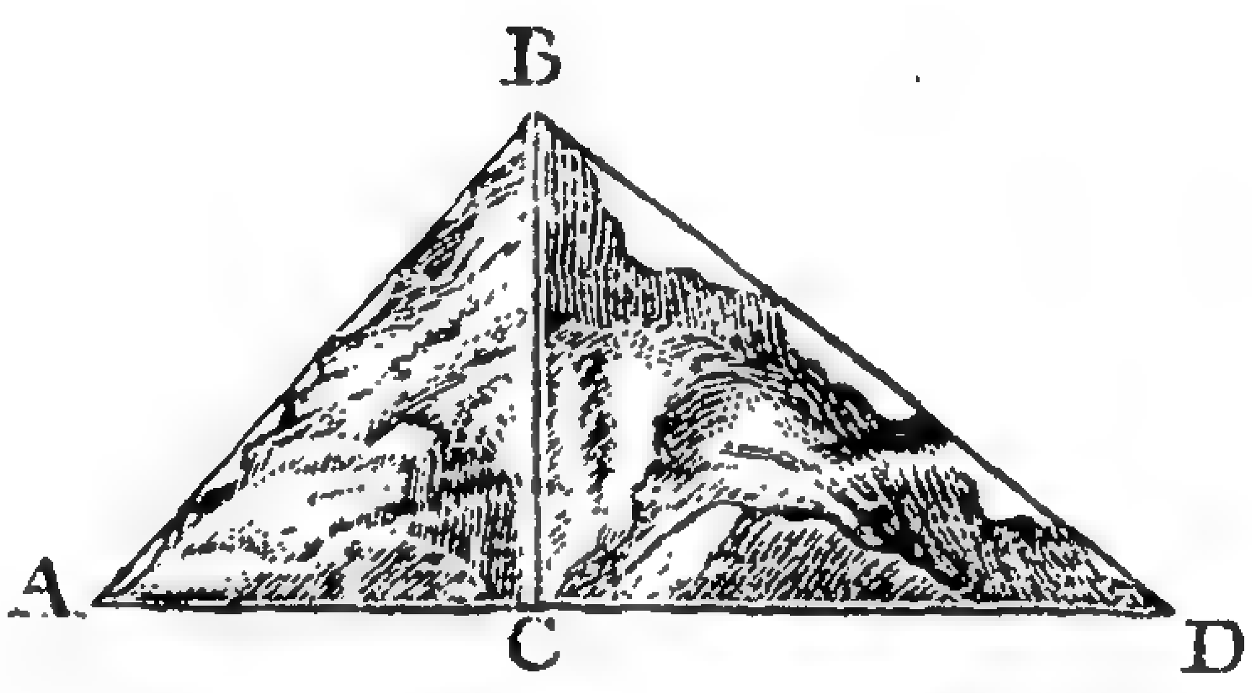
Let ABD be the *Hill.*—Firſt, ſet up a Mark on the Top at B, equal to the Height of the *Quadrant* or Inſtrument that is uſed at the Bottom, from whence you intend to make your Obſervation. Then by looking through the Sights to B take the Quantity of the Angle at A, which we will ſuppoſe to be 50°. Next meaſure the Hill from A to B, which let be 546 Feet. This being done, you may eaſily find the *Perpendicular* BC, or Part of the *Baſe* AC, by Caſe II. of Right Angles.

For the Perpendicular BC.

N. Rad. :	Side AB : :	Ang A :	Perp. BC
As 65.2 ——	546 ——	50 ——	418.7

For the Side AC.

N. Rad. :	Side AB : :	Ang. B :	Side AC
As 62.2 ——	546 ——	40 ——	351.1



Now, as the Hill *deſcends,* you may go on the oppoſite Side, and make the like Obſervations, viz. ſet up the Inſtrument at D, and take the Angle D, 40°, and meaſure the Side DB, 651 Feet, then you may find the Side CD in the ſame manner you did AC. Thus,

Nat. Rad. :	Side BD : :	Ang. B :	Side CD
As 65.23 ——	651 ——	50 ——	499

If to the Part AC = 351.1 Feet, be added the Part CD = 499 Feet, the Sum 850.1 Feet will be the whole Length of the *Horizontal Line* AD requir'd.

The *Perpendicular Height* DC is = 418.7 as above.

PROBLEM

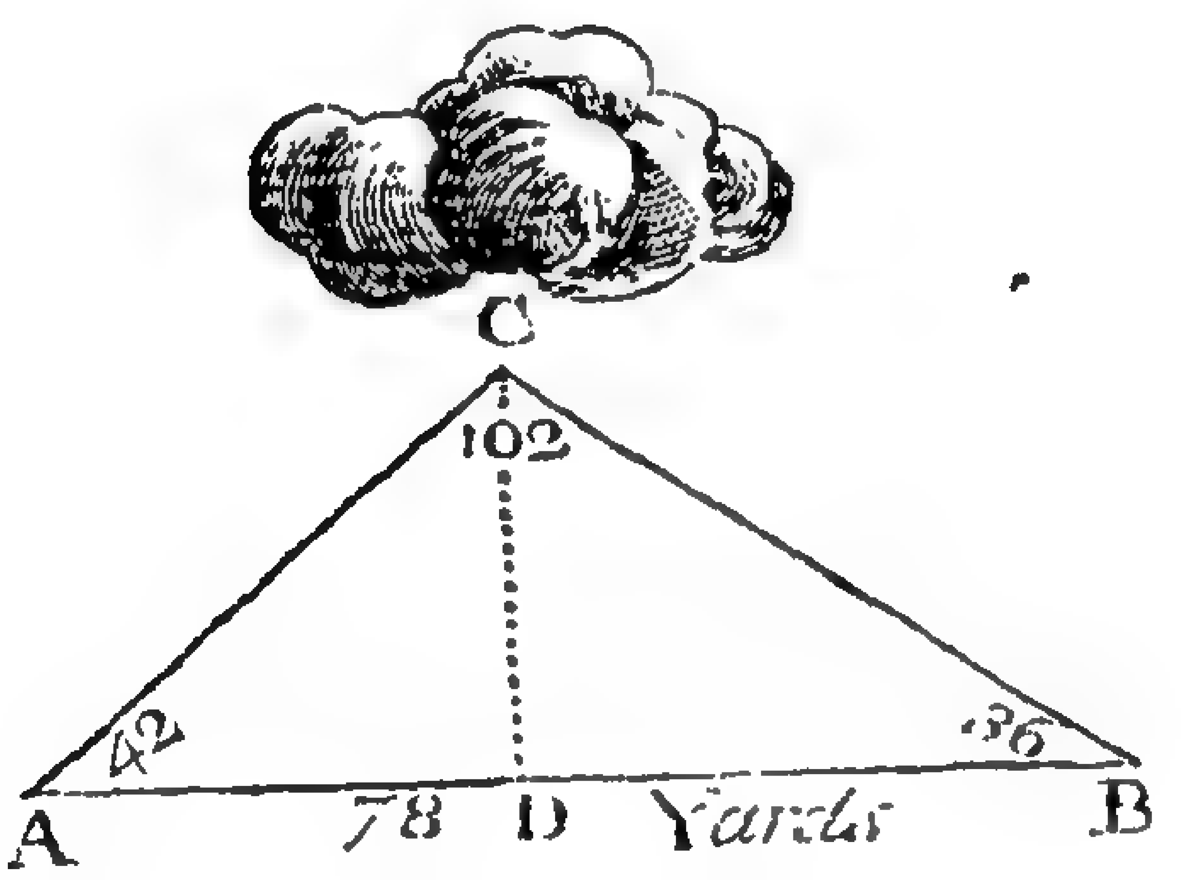
PROBLEM IX.

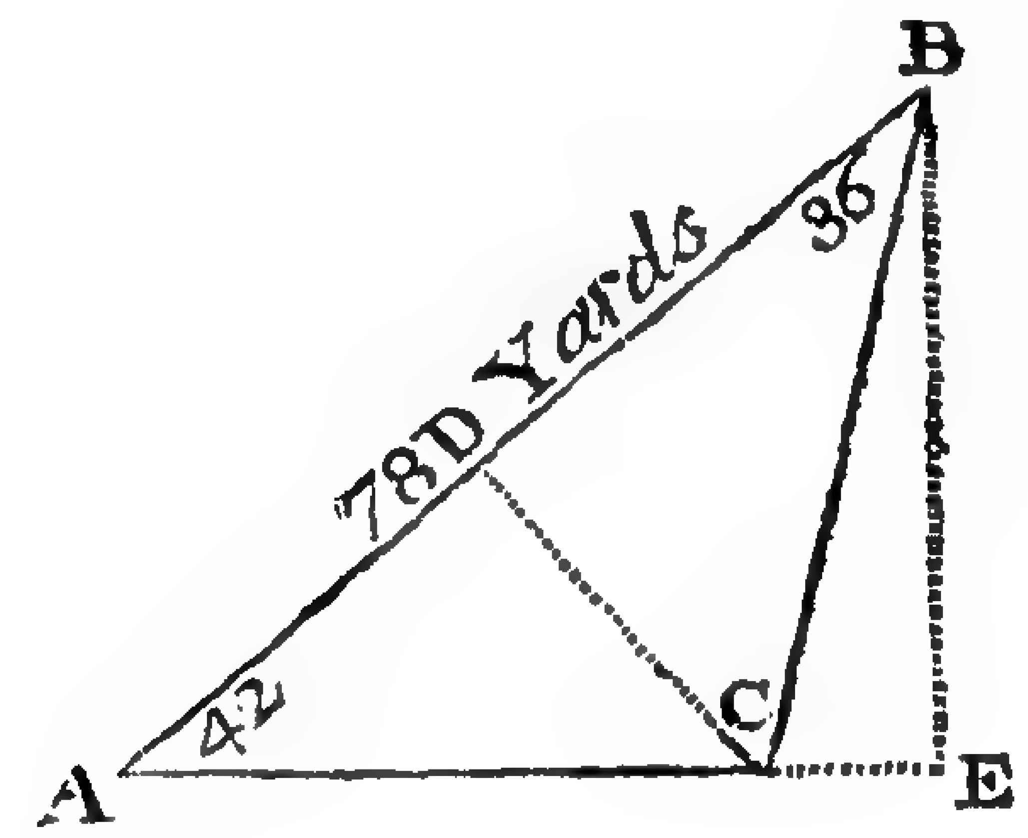
To take the *Height* and *Diſtance* of a *Cloud.*

Suppoſe it was required to find the Height of the *Cloud* C.

Let a Perſon ſtanding at A, look through the *Quadrant* to the *Cloud* at C, ſo will the Thread cut the Angle at A. At the ſame Time let another Perſon, making the like Obſervation at B, take the Angle B. Then meaſure the *Diſtance* between the *two Stations.* By this means you will have *one Side* and *all* the *Angles* of an *Oblique Angle Triangle* given, from whence you may eaſily obtain the reſt, and particularly the *Perpendicular* CD, which will be the *Height* of the *Cloud* required.

EXAMPLE. Suppoſe the Angle at A, by Obſervation, be 42°, the Angle B 36°, and the Diſtance AB 78 Yards: I demand the *Height* of the *Cloud.*





(1ſt.) Find the Perpendicular BE in Triangle ABE *.

N. Rad. : Op. Side AB. : : Ang. A : Perp. BE
As 62.6 —— 78 —— 42 —— 52.3

(2d.) Find the Hypothenuſe CB in Triangle CBE.

Ang BCE : Op. Side BE : : N. Rad. : Hyp. BC
As 78 —— 52.3 —— 79 —— 52.96

(3d.) Find the Perpendicular CD in Triangle CDB.

N. Rad. : Op. Side BC : : Ang. DBA : Perp. CD
As 61.1 —— 52.96 —— 36 —— 31.2

Anſwer, 31.2 Yards, the Height required.

* The Figure on the *Right Hand* is only that on the *Left* ſet in a different Poſition, to ſhew in a more natural or eaſy Manner, how the Perpendicular falls from the End of the given Side AB, upon the Side AC produced to E.

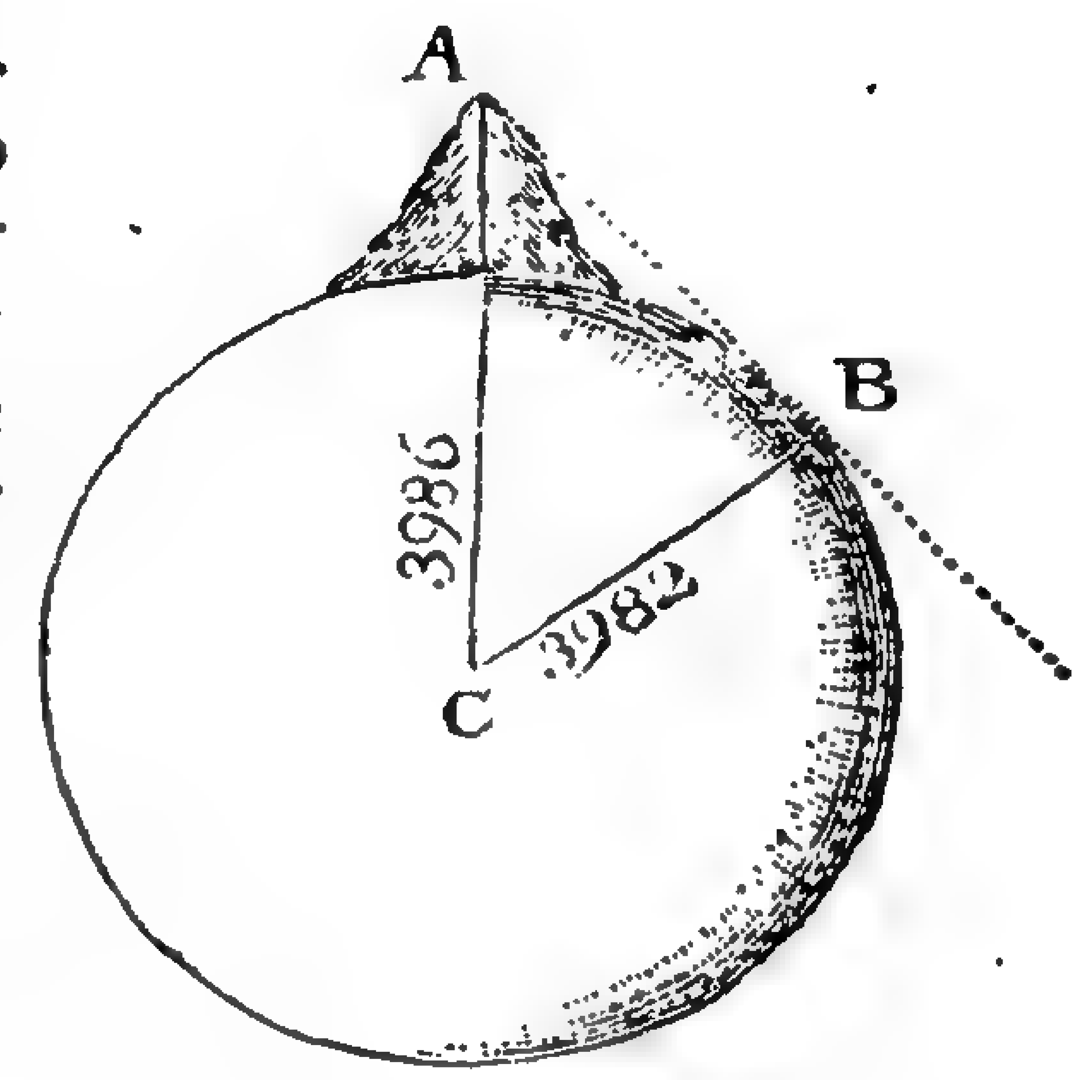
PROBLEM

PROBLEM X.

To find how far a *Hill* of any given *Height* can be ſeen at *Sea*, or upon *level Ground.*

How far, for Inſtance, can the *Pike* of *Teneriff* be ſeen at *Sea*, whoſe Height is about *four Miles.*

☞ The *Circumference* of the *Earth* is ſuppos'd by *Mathematicians* to be divided into 360 equal Parts, called *Degrees*; and our countryman, Mr. *Norwood,* has found, by meaſuring from the *Tower* of *London* to the Middle of the City of *York*, in the Year 1635, that one of thoſe Degrees, upon the Earth's Surface, contains $69\frac{1}{2}$ Miles; according to which Meaſure, we find the Earth's *Circumference* to be 25020 Miles—its *Diameter* 7964 — and its *Semidiameter* 3982.



Then in the Triangle ABC, Right Angled at B, we have the Side CB = the Earth's Semidiameter 3982. Alſo the Line AC = the Semidiameter and Height of the Mountain together = 3986. To find AB, the Diſtance from the Hill to the viſible Horizon.

```
   To Hypoth. AC  3986
     Add Leg BC   3982
                  ----
            Sum   7968
Multiply by Differ.  4
               ---------
Extract the Root 31872.00(178.5 The Diſtance the
                 1               Mountain can be ſeen.
                 --
              27)218
                 189
                 ----
             348)2972
                 2784
                 -----
            3565)18800
                 17825
                 -----
                   975
```

This *Mountain* can be ſeen 178.5 Miles at *Sea.*

PROBLE

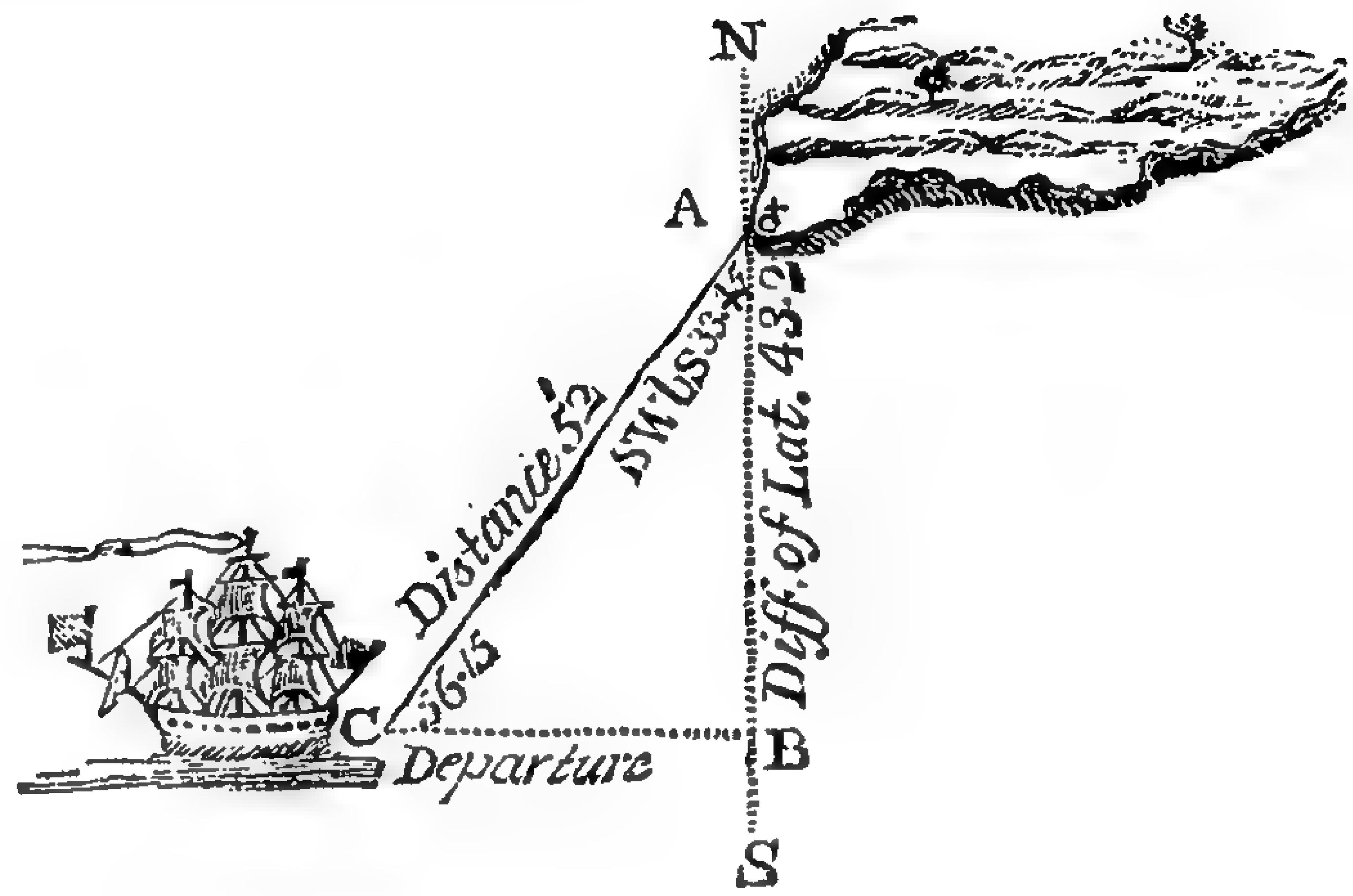
PROBLEM XI.

The *Distance run* at Sea, and the *Course*, given; to find the *Difference* of *Latitude* and *Departure* from the *Meridian*.

Suppose a *Ship* from A, in the *Latitude* of 50° North, sails away, SW by S. 52 Miles, to C: I demand the *Latitude* she is in, and also her *Departure* from the *Meridian*.

```
      33.75
      33.75
     ------
      16875
     23625
    10125
   10125
   -----------
   1139.0625
           3
   -----------
   3.417.1875
  57.3
```

Nat. Rad. 60.7 is enough.



(1st.) For the Departure.

```
  N. Rad.   Hyp. AC   ∠ A
As 60.7  —   52   —   33.75
                        52
                     -----
                      6750
                     16875
                    ------
            60.7)1755.00(28.9 The Departure
                 1214          CB
                 ----
                  5410
                  4856
                  ----
                   5540
                   5463
                   ----
                     77
```

```
Latitude departed from    50° North
Difference of Latitude     0   43.2
                          ---------
Latitude the Ship is in   49   16.8
```

(2d.) For the Diff. of Latitude AB.

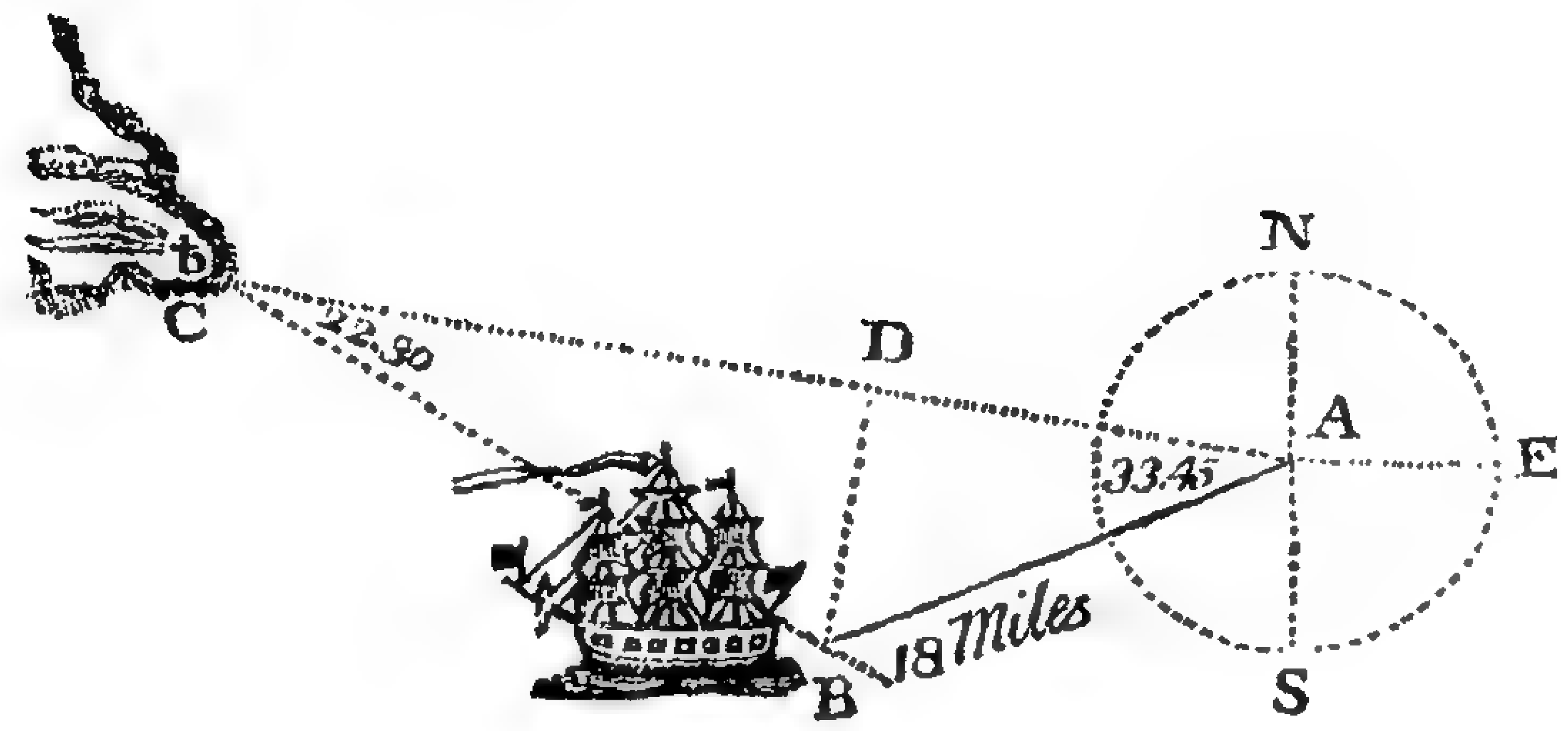
```
To Hypoth. AC      52
 Add Side BC       28.9
                   ----
         Sum       80.9
Multiply by Differ. 23.1
                   ----
                    809
                  2427
                 1618
                 ------
Extract the Root 1868.79(43.2  Differ. of Latitude
                 16
                 --
               83)268
                  249
                  ---
              862)1979
                  1724
                  ----
                   255
```

NOTE. That in all Cases of *Sailing*, we suppose the *Top* of the Book *North*, and *Bottom South*; the *Right Hand East*, *Left Hand West*. The *Distance run* is the *Hypothenuse*; the *Difference of Latitude* is the *Perpendicular*; the *Departure* the *Base*. The *Angle* at the *Perpendicular* is the *Course*, and the *other* its *Complement*.

PROBLEM XII.

To take the *Diſtance* of any *Cape, Fort,* or *Iſland,* from a *Ship* at *Sea.*

Sailing W. S. W. I ſaw, at ſome Diſtance, a Point of Land, which I *ſet*, and find it bears from me W. by N. and having ſailed 6 Leagues further, I find it then bears from me N. W. by W. I would know how far this Land is from me.



(1ſt.) Find the Perpend. BD in Triangle ABD.

N. Rad. Op. Side AB ∠ A
As 60.7 —— 18 — 33.75
18
27000
3375
60.7)607.50(10 Perpend.
607
.50

33.75
33.75
16875
23625
10125
10125
1139.0625
3
3.417.1875
57.3
Nat. Rad. 60.7 is enough.

(2d.) Find the Diſtance CB in Triangle BCD.

∠ C Op. Side BD Nat. Rad.
As 22.5 —— 10 —— 58.8
10
22.5)5880(26.13
450
1380
1350
300
225
750
675
75

22.5
22.5
1125
450
450
506.25
3
1.518.75
57.3
N. Rad. 58.8 is enough.

Anſwer, 26.13 Miles, the Diſtance required.

PROBLEM

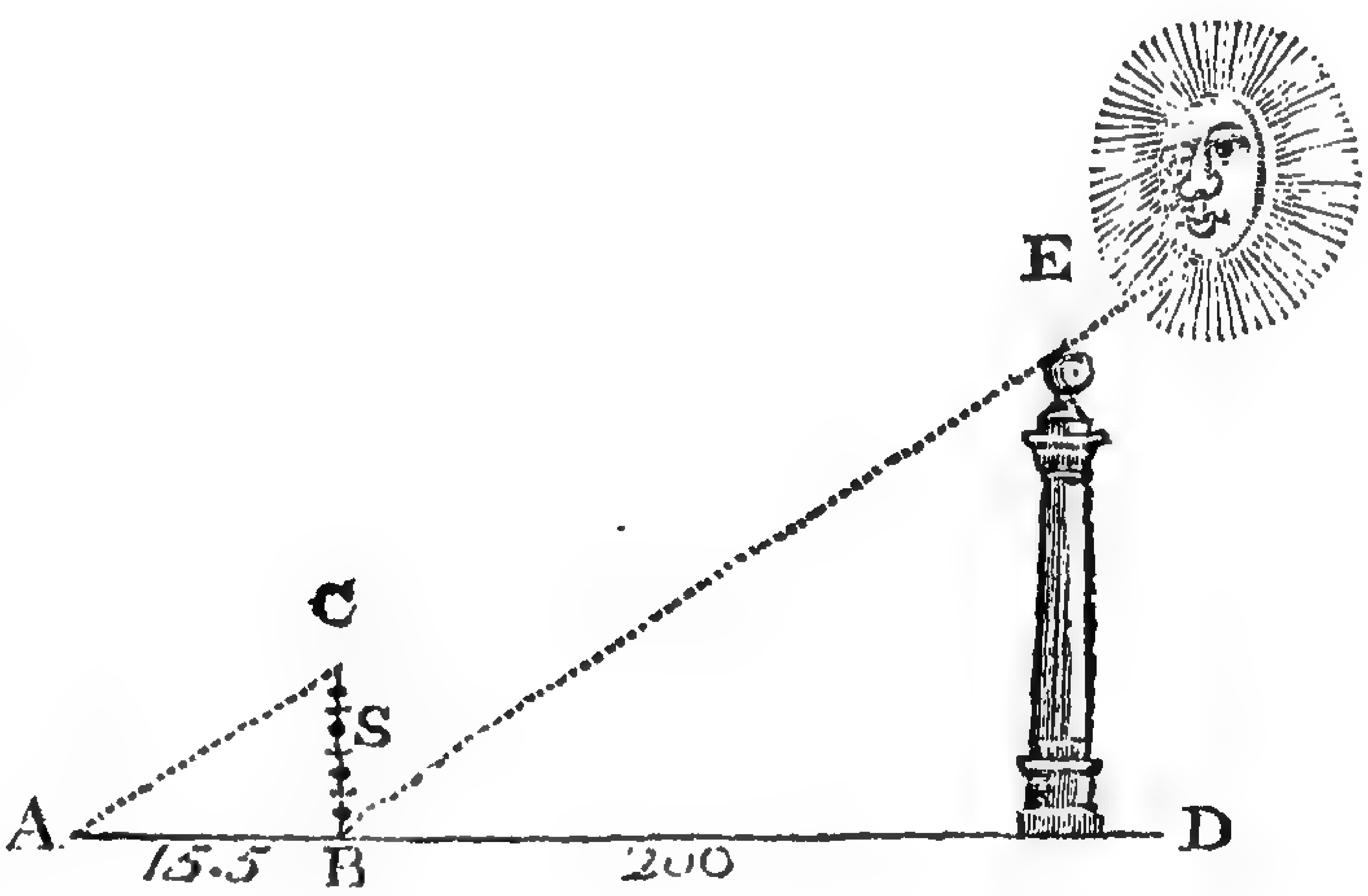
P R O B L E M XIII.

To take the Height of a *Tree, Fort, Obelisk, Pyramid,* or any *Object,* by a *common Stick* only, when the *Sun* or *Moon* shines upon it.

Take a Stick of any Length, suppose 8 Feet; set it upright upon the Ground, as at CB in the Figure below. Mark the End of its Shadow at A, and measure its Length from B to A, which suppose to be 15.5 Feet. Then measure the Length of the Shadow of the *Pillar* or *Obelisk* BD, which suppose to be 200 Feet. This being done, you may easily find the Height: For (by Reason of like Triangles) it will always hold,—as the Length of the Shadow of the Stick AB in the small Triangle, is to its Height CB; so is the Length of the Shadow of the Obelisk BD in the great Triangle, to DE the Height thereof.

Shad. St. Stick. Shad. Ob.
As 15.5 — 8 — 200
8

15.5)1600.0(103.2
155 . . Height requir'd
..500
465
.350
310
.40



In this *Manner* the Heights of the *Pyramids* in *Egypt* have been taken. Those stupendous Buildings are supposed to have been erected by the *Children of Israel,* when in Bondage, for *Sepulchers* for the *Egyptian Kings.* They are the greatest Pieces of *Antiquity* now in Existence. There are several *smaller Ones,* but the *largest,* which is justly esteemed one of the *Wonders of the World,* is 500 Feet in *Perpendicular Height;* —700 Feet if measured *obliquely* from the Bottom to the Top;—and its *Base* covers about 11 *Acres of Ground.*

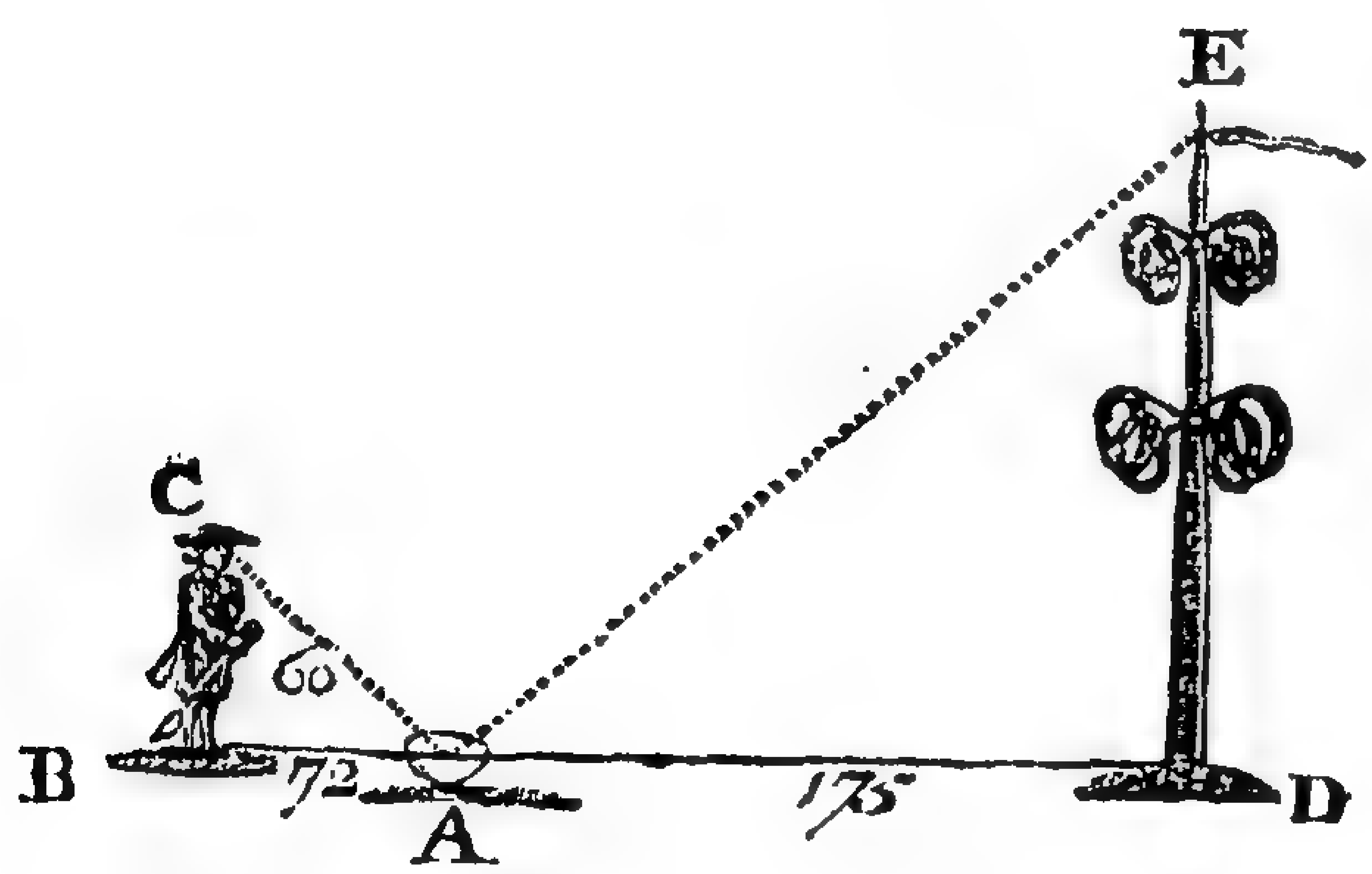
PROBLEM

PROBLEM XIV.

To take the *Height* of any *acceſſible Object* by a *Baſon of Water*, or common *Looking Glaſs*.

Travelling along the Road, I ſee a fine *May Pole*, whoſe Height I would gladly know; but having no *Mathematical Inſtrument* with me, I procure a *Baſon of Water*, which I ſet upon the Ground, at ſome Diſtance from the Pole, as at A; then I go backwards, till I ſee the Top of the Pole in the Middle of the Water, as at B. This done, I meaſure the Diſtance from my Station at B to the Baſon at A, which ſuppoſe 72 Inches; and alſo meaſure from the Baſon to the Bottom of the Pole at D, and find it 175 Inches. Next I meaſure the Height of the Eye from the Ground, which ſuppoſe 60 Inches. Then ſay, by the *Rule of Three*,

As the *Diſtance* from my *Station* to the *Baſon*,	= 72	Inches.
Is to the *Height* of the *Eye*,	= 60	
So is the *Diſt.* from the *Baſon* to the *Foot* of the *Pole*,	= 175	
To the *Height* of the *Pole* requir'd,	= 145.8	



The ſame Thing may be obtain'd by a *Looking Glaſs*, laid truly *Horizontal*, or level on the Ground, by walking back till you can ſee the *Top* of the *Building*, &c. in the *Middle* of it, as was done by the *Water*.

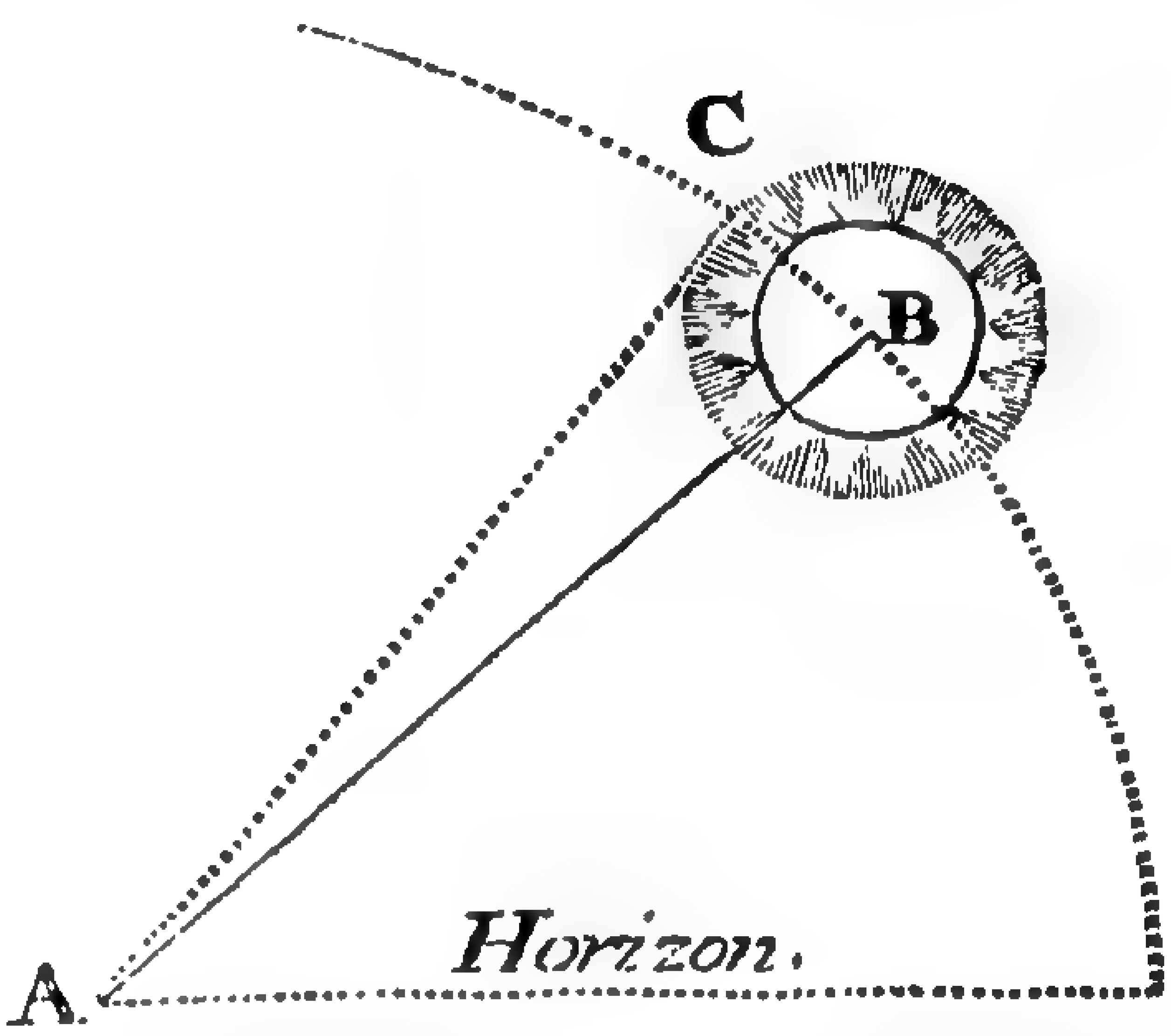
PROBLEM

PROBLEM XV.

To take the Diſtance of the *Sun, Moon,* or any of the *Heavenly Bodies.*

Suppoſe it was requir'd to find the Diſtance of the *Sun,* in *Diameters* of his Body from us.

With a *Quadrant* nicely graduated, take the *Altitude* of the *lower* and *upper Limb* in *Degrees* and *Minutes,* and ſubtract the one from the other; the Remainder will give the *Diameter* of the *Sun,* which we will ſuppoſe, in this Caſe, to be 32 Minutes.



Then have we given in the Triangle ABC, Right Angled at B,—the Angle at A = 16′, and the Side CB = .5 = Half the Diameter of the Sun, whoſe whole Diameter we will call 1; to find the Diſtance, or Side AC.

```
   Ang. A  :  Side BC : : Nat. Rad.
As .26  ——   .5   ——   57 3
                         .5
                       ————
             .26)2865(110 Diameters; and ſo far is the Sun of its own Breadths from us in the
                 26           Winter *, but in Summer, the Angle being a little ſmaller, he
                ———           muſt, conſequently, be a little further from us.
                  26
                  26
                 ———
                   05
```

Having found the *Diſtance* of any *Heavenly Body* in its own *Diameters* from us; you may eaſily tell its *Diſtance* in *Miles,* if you firſt know the *Diameter* of that Body in *Miles.* For, the Diſtance in Diameters, multiply'd by the Miles in one Diameter, gives the Diſtance ſought.

For the Uſe of the Learner, I have here ſubjoined a Table of the *Diameters* of all the *Planets* in *Engliſh Miles*; whoſe *Diſtances* he may calculate at his Leiſure.

Sun 800.000---*Mercury* 2460---*Venus* 7906---*Earth* 7970----*Mars* 4444 ---*Jupiter* 81.155---*Saturn* 67.870---*Moon* 2175.

* In this Manner we can tell the *apparent* Diſtance of any of the *Heavenly Bodies:* For the *Sun* appearing about 1 *Foot* in Diameter, his *apparent* Diſtance can be only 110 *Feet,* or 37 *Yards.* The *apparent* Diſtance of the *Moon* is nearly the ſame.

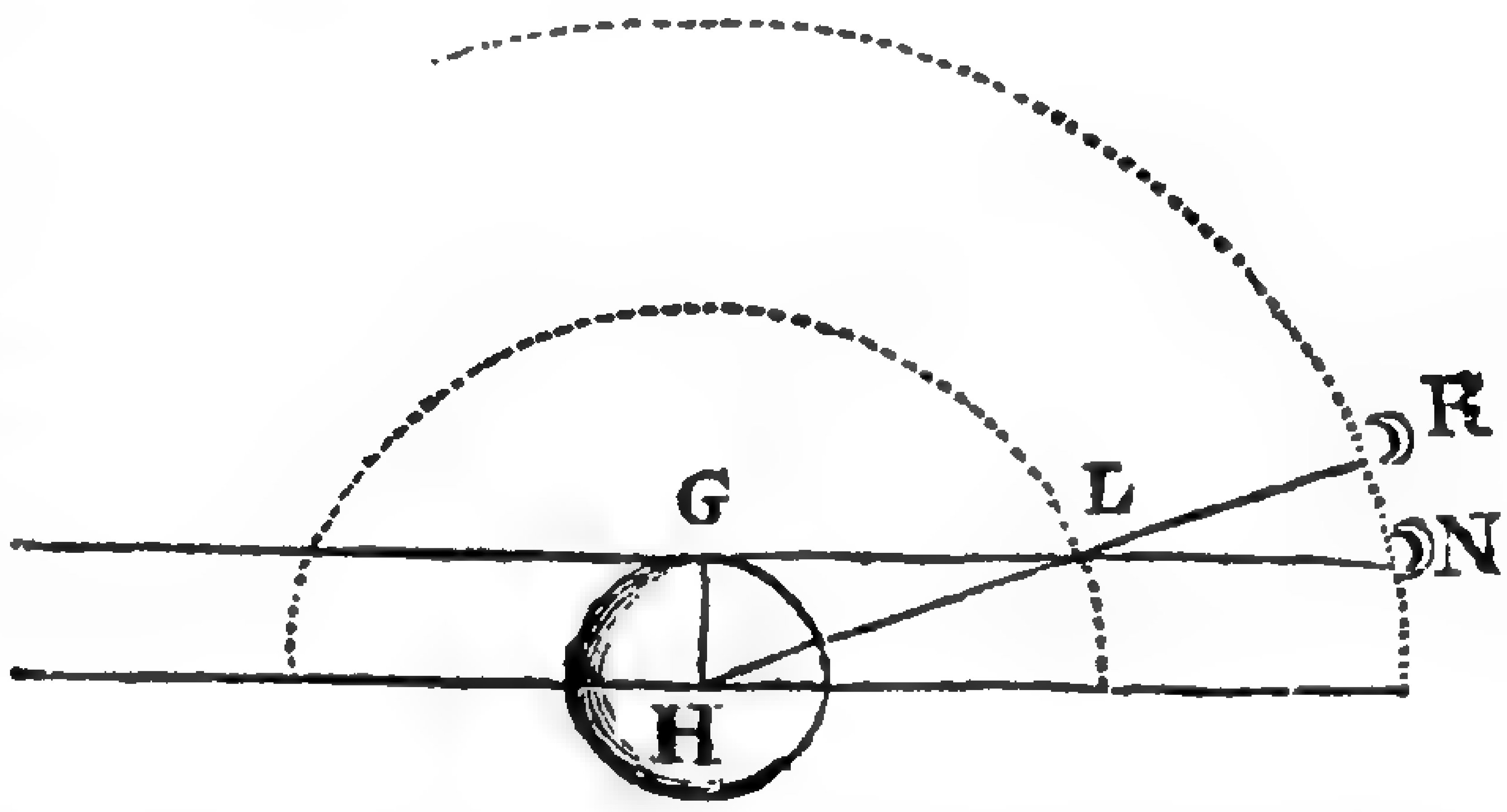
PROBLEM XVI.

To find the *Distance* of the *Moon* from the *Earth* another Way.

As the Places of all the *Heavenly Bodies* are computed from the *Center* of the *Earth*, but we are obliged to view them from the *Superfices*, they must therefore appear something lower in the Heavens than they really are.

Thus, suppose, for Instance, the ☽ to be at L in the visible Horizon; an Observer at G will see her in the Line GN, but an Eye in the Center will see her in the Line HR.---The former is her *apparent Place*, known by Observation with exact Instruments; the latter her *true Place*, and known by *Calculation* from *Astronomical Tables* of her Motion. The Difference between these two Places is the Measure of the Angle GLH, which is called the *Horizontal Parallax*. This Angle has been found, when the ☽ was at a *mean Distance*, to be 57 Minutes nearly.

Then in the Triangle HGL, Right Angled at G, we have the Angle at L = 57′, the Side GH = the Earth's *Semidiameter* given; to find HL, which is done thus.



Ang. L : Semid. GH : : N. Rad.
As .95 —— 1 —— 57.3
1

.95)57.30(60 Semidiameters; which multiply'd by the Earth's Semidiameter = 4000 nearly, you have 240000 Miles the Distance of the Moon sought.
570
——
30

When the ☽ is at her *least* or *greatest* Distance from the Earth, she will be about 3 *Semidiameters* of the Earth *nearer* or *farther off*, than at her *mean Distance*.

In like Manner may the Distance of any of the *Planets*, *Comets*, or other *cœlestial Phænomenon* be determined, by obtaining its *Parallax* in the *Horizon*.

NOTE. To find the *Distance* of any Place to which the *Sun*, *Moon*, or any *Star* is *vertical*, i. e. over its Head, or in its *Zenith*. See my *Geography*, p. 30.

PROBLEM XVII.

To determine the *Diſtance* of the *Sun*, and all the *Planets*, more accurately than in the laſt Problem.

The *Horizontal Parallax* of the *Moon* being very difficult, if not impoſſible, to determine with Accuracy, on Account of the Uncertainty and Mutability of the Horizontal Refractions, which are varying according to the State of the *Atmoſphere:* Aſtronomers have purſued other Methods of doing it. That which ſeem'd to bid the faireſt —was to determine the Parallax of *Venus*, at a Time that, that Planet TRANSITS the *Sun's Diſk*. This Method was firſt propos'd by Dr. Halley, and many accurate Obſervations were made in diſtant Parts of the Globe, according to his Propoſal. The Reſult of the ſeveral Obſervations made here and abroad was—that the *Parallax of the Sun* on the Day of the *Tranſit*, June 6, 1761, was 8″.52. At which Time the Sun was nearly at his greateſt Diſtance from the Earth: Conſequently, his Parallax at his *mean Diſtance* will be ſomething more, viz. 8″.65.

Let S repreſent the *Sun*, E the *Earth*; then in the Triangle SCD Right Angled at C; theſe are given the Angle at S = 8″.65, (the Angle under which the *Semidiameter* of the

Earth appears (at that Time) at the *Sun*,) and the Side CD = the *Earth's Semidiameter*;—to find the *Diſtance* SD or SC. Thus,

	Angle S	: Side CD	:: Nat. Rad.	: Side SD	
As,	0024	—— 1	—— 57.3	—— 23875	Diſtance of the Sun in Semidiameters of the Earth,

which multiplied by the Earth's Semidiameter gives upwards of 95 Millions of Miles.—The *Sun*, and conſequently *all the Planets*, by this Obſervation of the Tranſit of *Venus* are found to be further off, than by all former Obſervations, about $\frac{1}{6}$ Part or ſomething more.

The Table at Page 33 of the View of the Heavens, exhibits the Diſtances of the *Planets*, according to the *late* and *preſent Calculations*.

☞ The *Diſtance* of the neareſt *Fix'd Star* is ſo immenſe, that it *cannot be aſcertain'd by this Method.*

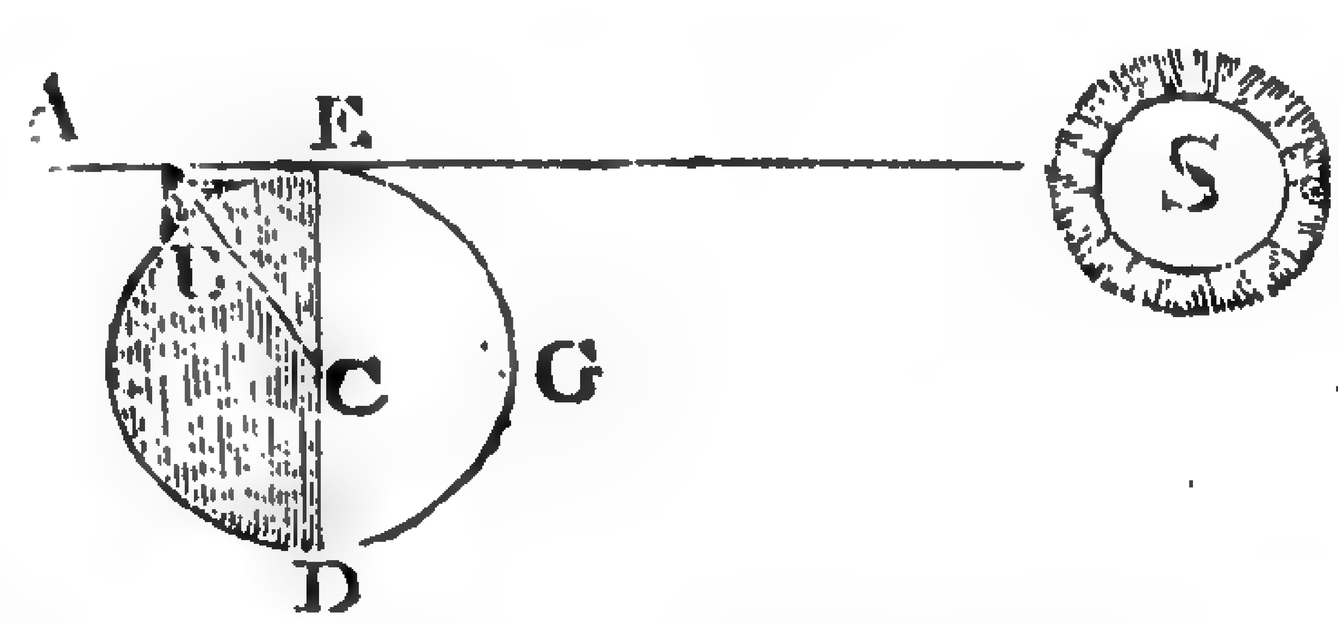
PROBLEM

PROBLEM XVIII.

To meaſure the *Height* of a *Mountain* in the *Moon*.

The *Moon* is ſuppos'd to be compos'd of *Land* and *Water* as our Earth is; conſequently there muſt be ſome Unevenneſſes, or Inequalities, of *Hills* and *Vallies* as here. This indeed is confirmed by viewing her thro' a good *Teleſcope*; for then we find, that the Line, which ſeperates the Light from the Dark Parts on her Surface is not even or regular, but tooth'd and jagg'd with innumerable Breaks; and even in the Dark Parts, near the Borders of the lucid Surface, there are ſeen ſome ſmall Spots enlighten'd by the Sun, which are very viſible when the ☽ is three or four Days old, and which can be nothing elſe but the Tops of *Mountains* or *Rocks*; ſince it is impoſſible for the Sun's Rays to fall upon thoſe Parts only, unleſs they were higher than the Reſt of the Surface.

The *Lunar Mountains* are found to be *higher*, in Proportion to the Body of the ☽ than any Hills upon our Globe. The Manner of calculating their Heights is this.



Let EGD be the Surface of the ☽ and ECD the Diameter of the Circle bounding *Light* and *Darkneſs*. A the Top of a Hill within the dark Part, when it firſt begins to be illuminated by a Ray of Light coming from the Sun at S. Then obſerve with a Teleſcope the Proportion of the Right Line AE, (*i. e.* the Diſtance of the Point A from the lucid Part) to the Diameter (or Semidiameter) of the ☽ ED, for that being aſcertain'd, you have in the Triangle AEC, Right Angled at E, (where the Ray of Light touches the ☽,) the two Sides AE and CE, to find the *Hypothenuſe* AC, from which ſubtracting BC = EC, there will remain AB the Height of the *Mountain*.

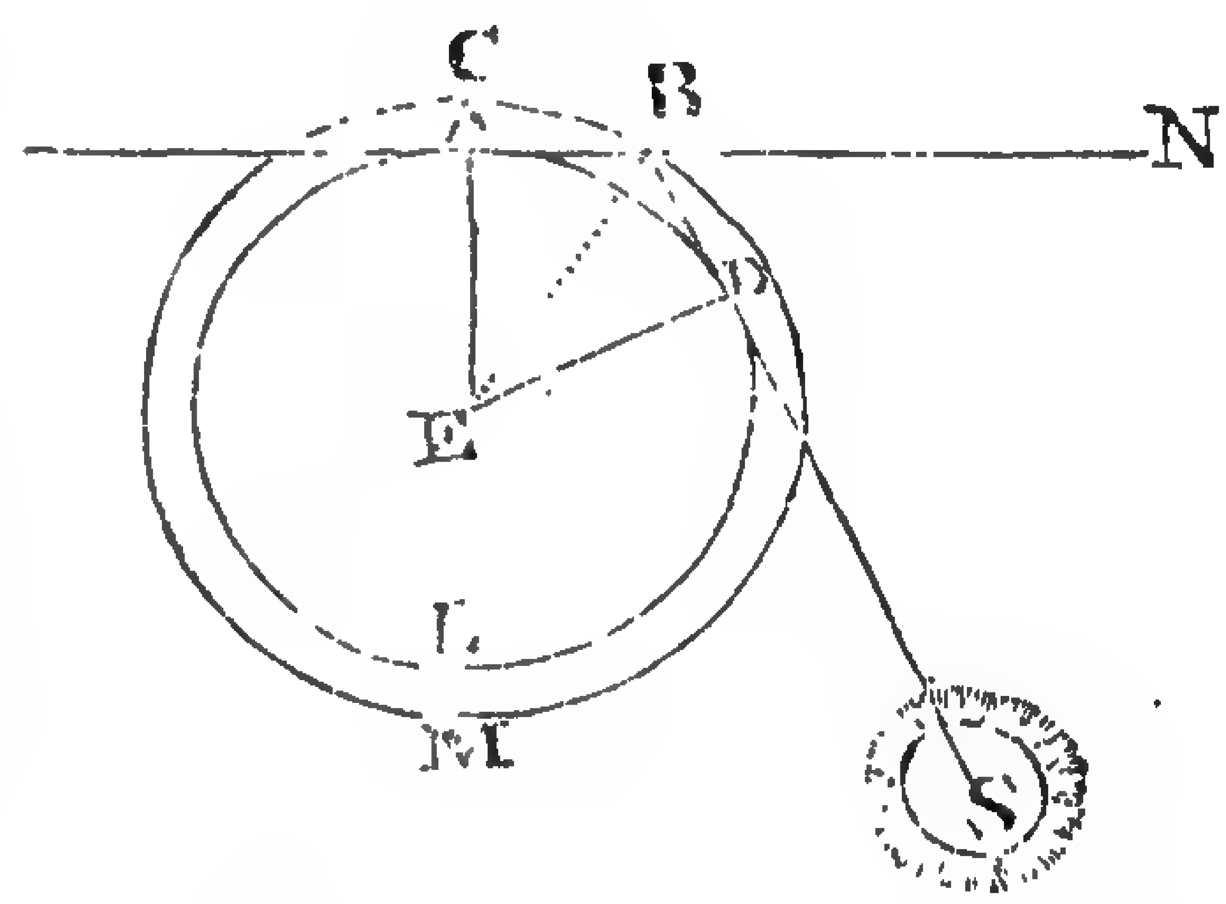
Ricciolus, on viewing the ☽ when about four Days old, obſerv'd the Top of a Hill called *Saint Catherine*, near the N. Part of *Mount Taurus*, (ſee my *Aſtronomy*) to be illuminated, and that it was then diſtant from the Surface about $\frac{1}{8}$ of the Moon's *Semidiameter*. Now as, the *Semidiameter* of the ☽ EC is about 1088 Miles, the Line AE being $\frac{1}{8}$ of it, muſt be = 136 Miles. Conſequently, if the □ of EC and □ of AE be added together, and then the □ Root of it be extracted, it will give the Line AC, from which ſubtracting the Moon's *Semidiameter* BC or CE, the Remainder, which is 8 Miles will be the Height of the *Mountain* ſought.

PROBLEM

PROBLEM XIX.

To meaſure the *Height* of the *Atmoſphere*.

The *Atmoſphere* is that Circle of *vaporous Air* ſurrounding the Earth, which being illuminated by the *Sun's* Rays makes the Brightneſs and Glory of the *Firmament* we behold, whilſt the Sun continues above the *Horizon*. And, after the Sun is gone down, the *Atmoſphere*, which is higher than we are, will ſtill continue to be illuminated by thoſe Rays paſſing by the Earth over our Heads; but this *Brightneſs* grows *leſs* and *leſs* as the Sun deſcends lower, till he arrives at 18° below the *Horizon*; when all the Parts of the Air above fall out of his Rays, and, conſequently, become dark.



To make this plainer; ſuppoſe the inner Circle ADL repreſents the Earth, and the Circle CBM the *Atmoſphere*. Suppoſe a Perſon ſtanding upon the Earth at A whoſe *ſenſible Horizon* is AN. Alſo let SB be a Ray of Light coming from the *Sun*, touching the Earth at D, which falls upon the diſtant Part of the Air, in the Horizon at B, at which time *Twilight* ceaſes. This has been found to happen when the Sun is deſcended 18° below the *weſtern* Horizon in the *Evening*; and alſo when he is approached within 18° of the *eaſtern* Horizon in the *Morning*.

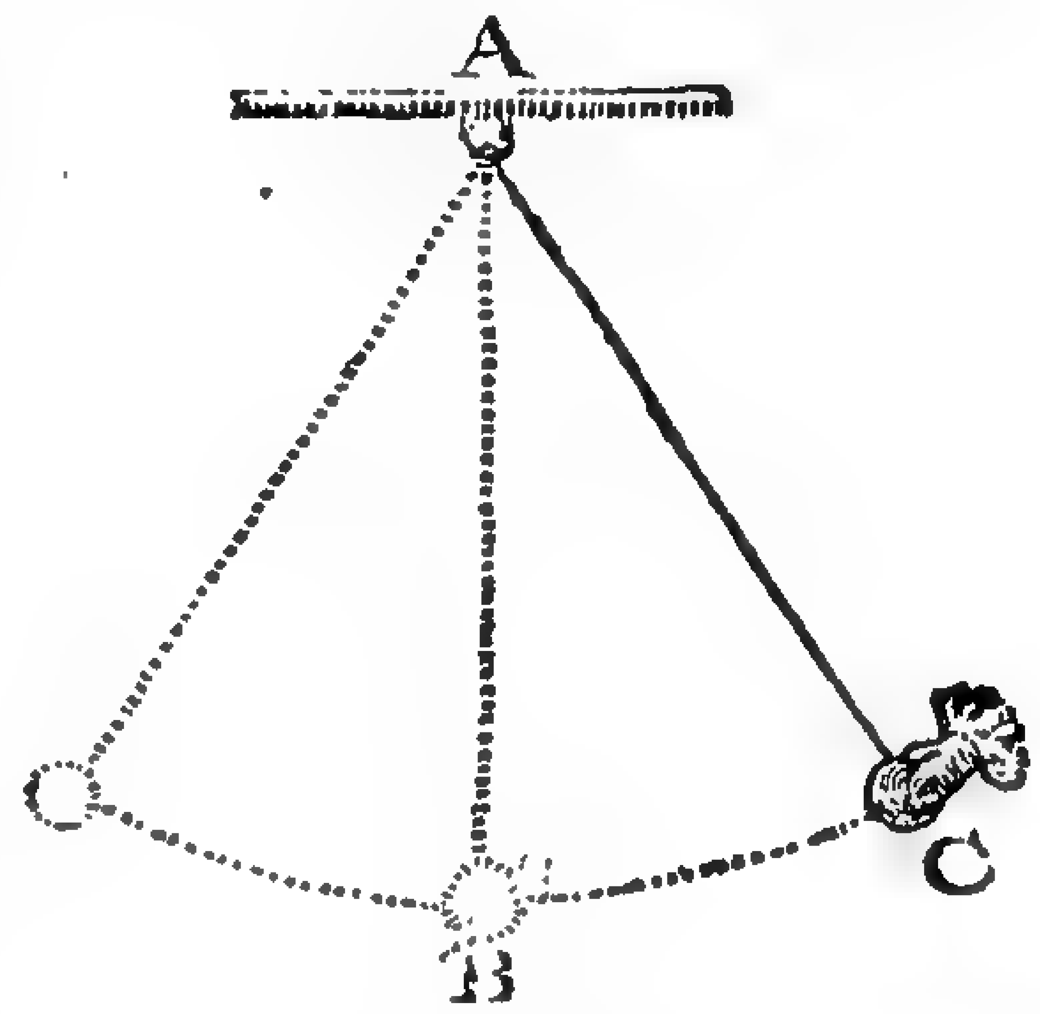
Now as the Arch AD is 18°, we have, by drawing the Line EB, two equal Triangles, from either of which we may find the Height of the *Atmoſphere* requir'd.—For in the Triangle ABE, Right Angled at A, we have given the Side AE the *Earth's Semidiameter*, and the Angle AEB = 9° = half the Arch of the *Sun's* Deſcent below the *Horizon*, and the Angle ABE = 81°, to find the *Hypothenuſe* EB, from which if you ſubtract the *Semidiameter* of the Earth, the Remainder will be the Height of the *Atmoſphere*.

Thus, Ang. B : Earth's Semid. AE :: N. Rad. : Hypoth. EB
As 81 —— 4000 —— 81.99 —— 4048
4000 Earth's Semidiameter ſubtract.
48 Height of the Atmoſphere requir'd.

PROBLEM XX.

To meaſure the *Diſtance* of a *Cloud,* from which iſſues *Lightnings* and *Thunder.*

Take a ſmall *Ball* of *Lead, Ivory,* or any other matter, and affix it to the End of a fine *Thread.* Then meaſure from the Center of the Ball, along the Thread, exactly 39.2 *Inches,* where make a *Loop.* This done, ſuſpend it by that Loop to the Ceiling of the Room, or to any other Place where it may hang freely, and vibrate backwards and forwards like a *Pendulum,* as in this *Figure.* Now the Property of this little Inſtrument is, that *each* Vibration, whether it paſſes through a larger or ſmaller Space, will be performed in *one Second* of Time.



Being thus prepar'd, take the Ball in your Hand, and drawing it aſide from its perpendicular Direction AB, to any Diſtance, ſuppoſe to C, hold it there till you ſee the flaſh of *Lightning* paſs by, at which Moment let it go, and count the *Number of Vibrations* till you hear the Stroke of the *Thunder.* Then theſe Vibrations multiplied by 1142, (the Number of Feet, *Sound* uniformly paſſes through in each *Second*) the Product will be the Height of the *Cloud* in Feet, if it *be nearly over the Place where you are*; or its Diſtance from you in any other *Situation.*

Thus, ſuppoſe the *String* is found to make 8 Vibrations between the *Lightning* and the *Thunder*; then $8 \times 1142 = 9136$ Feet, which, divided by 5280 (the Feet in 1 Mile) gives $1\frac{3}{4}$ Mile nearly; and ſo far is that alarming Tempeſt from you.

In this Manner you may continue to meaſure the Diſtance of the *Cloud* all the Time it paſſes from your *Zenith* to the *Horizon,* and by that Means be acquainted with the *Danger* it ſeems to threaten the *Neighbourhood,* as well as the *Extent* of the *viſible Hemiſphere* of *Clouds.*

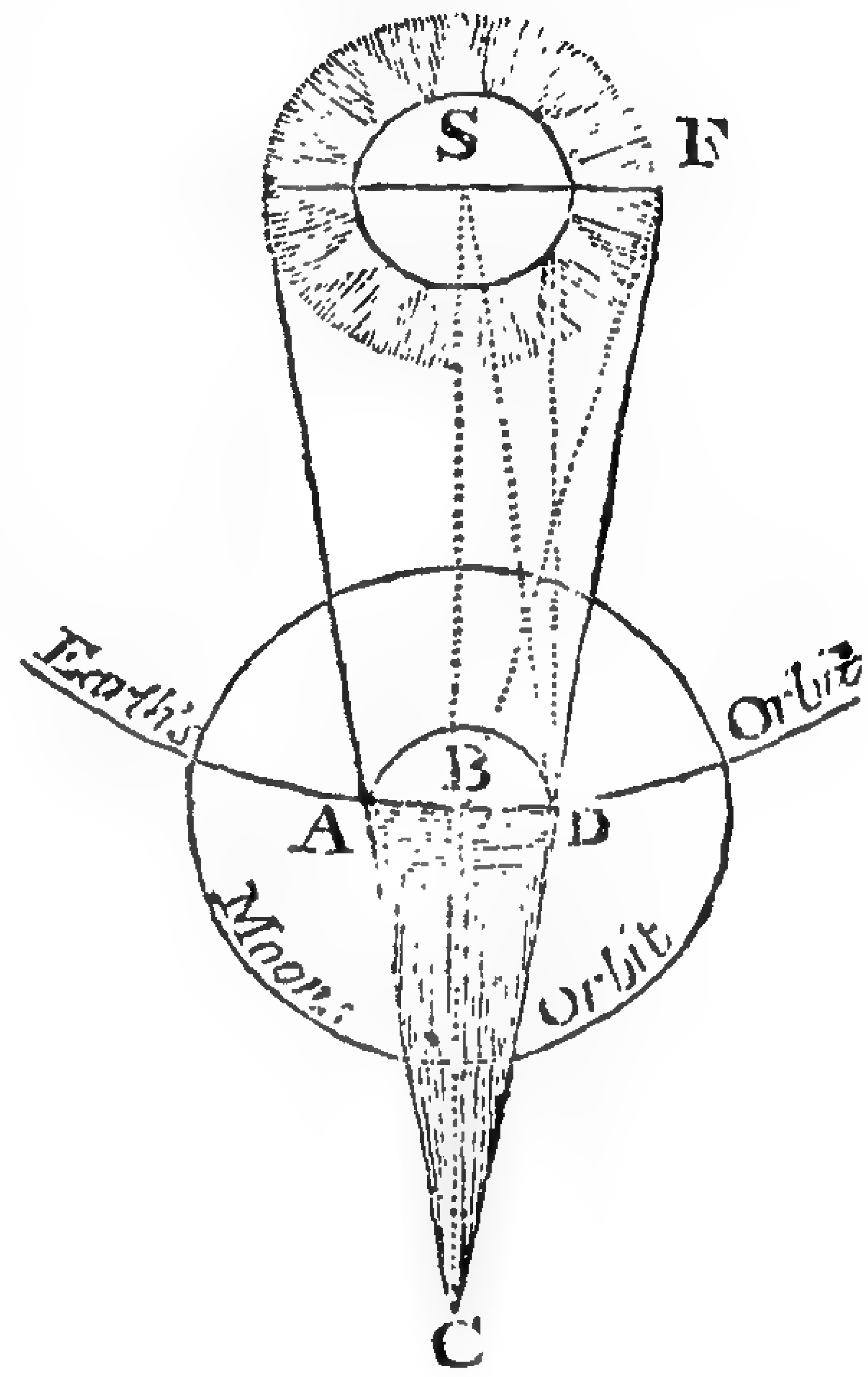
The *Diſtance* alſo of a *Ship* at *Sea,* or a *Fort,* may be eſtimated in the ſame Manner, by counting the Vibrations from the *Flaſh of the Powder* to *the Report of the Gun.*

PROBLEM

PROBLEM XXI.

To calculate the *Length* of the *Earth's* or *Moon's Shadow.*

The Angle of the *Cone* ACD of the *Earth's Shadow* in the annex'd Figure, is equal to the *Sun's* apparent *Diameter* *, which, at a mean Diſtance from us, is about 32 *Minutes.* Hence, in the Triangle ACB, Right Angled at B, we have the Angle ACB = 16 *Minutes,* the apparent *Semidiameter* of the *Sun,* and AB the *Semidiameter* of the *Earth* = 1; to find AC or BC the Length of the *Shadow,* which is done thus.



∠ ACB : Side AB :: N. Rad.
As .266 — 1 —— 57.3
1

.266)57.300(215 Semidiameters = AC
532

410
266

1440
1330

110

Thus, when the *Sun* is at a *mean* Diſtance from us, the *Shadow* of the *Earth* reaches about 215 *Semidiameters* beyond it: But when the *Sun* is at his *greateſt* or *leaſt* Diſtance, the *Shadow* will be *lengthened* or *ſhortened* 3 or 4 *Semidiameters,* more or leſs.

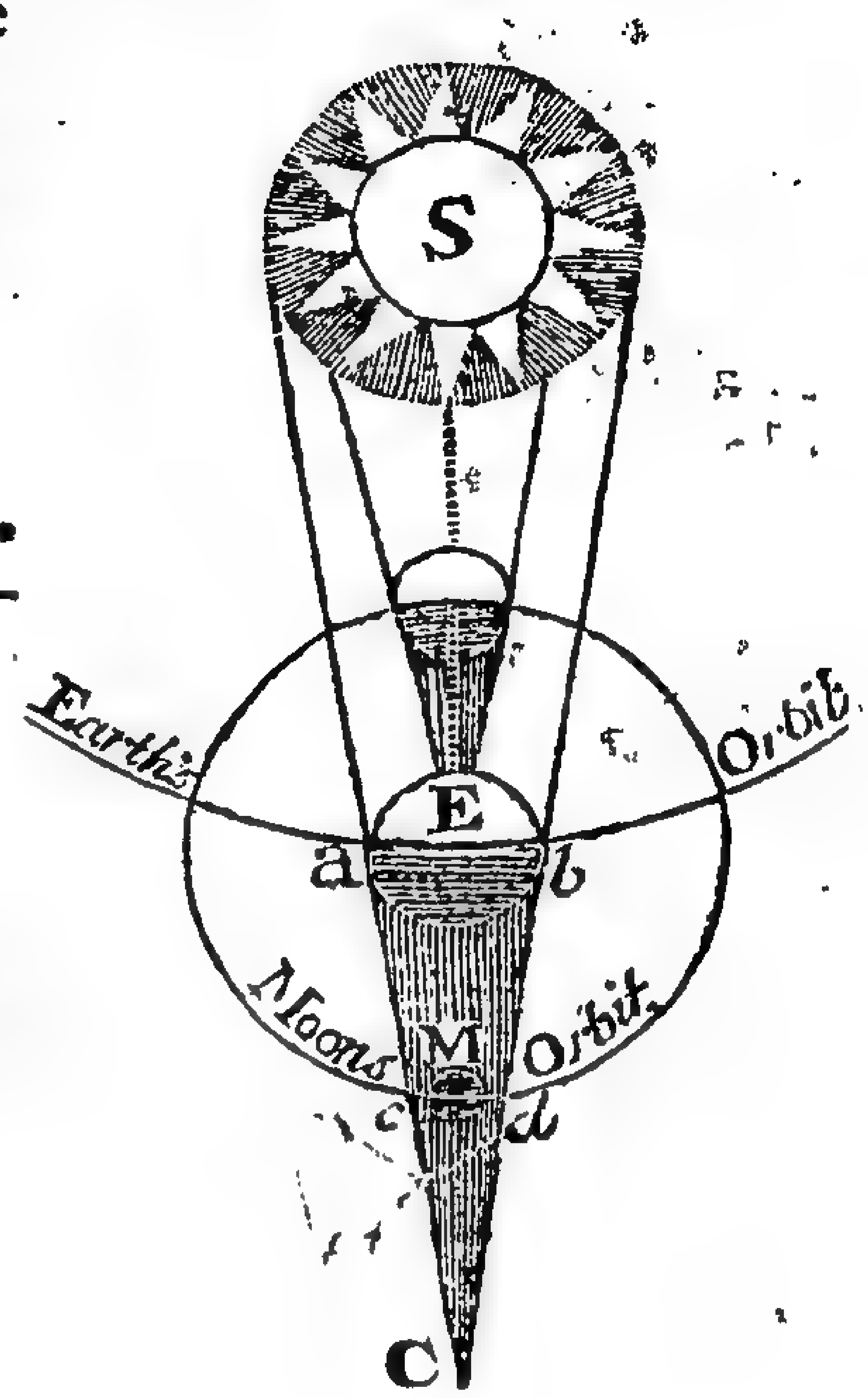
Hence, you may alſo determine the *Height* of the *Moon's Shadow:* For, as the *Moon* is never at any great Diſtance from the *Earth,* the apparent *Semidiameter* of the *Sun* muſt be nearly the ſame *there* as *here.* Conſequently, the *Moon's Shadow* muſt contain the ſame Number of *Semidiameters* of the *Moon,* as the *Earth's Shadow* does *Semidiameters* of the *Earth:* Which *Semidiameters,* multiply'd by the *Miles* in the *Semidiameter* of the *Moon* or *Earth,* will give the Length of the *Shadow* reſpectively in Miles.

* The *Semiangle* of the *Cone* of the *Earth's Shadow* BCD, is equal to the apparent *Semidiameter* of the *Sun* view'd from the Top of the Shadow, which Angle is always equal (in the Shadow of every Planet) to the apparent *Semidiameter* of the *Sun* SBF, leſſen'd by his *Horizontal Parallax* BSD at that Planet. But as the *Horizontal Parallax* of the *Sun, i. e.* the Angle under which the *Earth* is ſeen from thence, is ſcarcely 10 *Seconds,* it may be omitted, as is done in the above Calculation.

PROBLEM XXII.

To calculate the *Diameter* of the *Earth's Shadow* at the Diſtance of the *Moon*; and alſo, the *Diameter* of the *Moon's Shadow* at the *Earth*.

In the following Figure let S repreſent the *Sun* E, the *Center* of the *Earth*, M the *Moon*, EC the *Cone* of the *Earth's Shadow* (at a mean) = 215 *Semidiameters* of the *Earth*: Then MC will be the *Cone* of the *Earth's Shadow* reaching beyond the *Moon*, whoſe Length is thus found.



		Semidrs.
From EC the Cone of the Earth's Shadow -	=	215
Subtract EM the Diſt. of the Moon in the Earth's Semid.	=	60*
Remains MC the Shadow of the Earth beyond the Moon	=	155

Then, by Reaſon of ſimilar Triangles, it will always hold;

As the Length of the whole Shadow -	EC
Is to the Diameter of the Earth - -	ab
So is the Length of the Shadow beyond the Moon	MC
To the Diameter of the Shadow at the Moon	cd

	EC	:	ab	: :	MC
As	215	—	7964	—	155

```
   7964
    155
  -----
  39820
 39820
7964
-------
215)1234420(5741 Miles = cd, the Diameter of the Earth's Shadow at the Distance of the Moon.
    1075
    ----
     1594
     1505
     ----
       892
       860
       ---
        320
        215
        ---
        105
```

By this Problem alſo may be found the *Diameter* of the *Moon's Shadow* at the Surface of the Earth, and, conſequently, how much of the Earth is involv'd in that *Shadow* in an *Eclipſe* of the *Sun*. For the *Length* of the *Moon's Shadow* is found to be about 60 *Semidiameters* of the Earth, which is nearly the *Moon's mean* Diſtance from us; her *Shadow*, in that State, muſt, therefore, reach as far as the Center of the Earth.—But as the *Moon* is, ſometimes, almoſt 4 *Semidiameters* of the Earth *nearer*, the *Shadow* muſt reach 4 *Semidiameters* beyond the Center of the Earth; and when the *Moon* (as ſhe ſometimes is) is 4 *Semidiameters further* from us, the *Shadow* will then not reach the Earth at all. In ſuch Caſe, the *Sun*, though centrally *eclips'd*, will not be *totally* covered by the *Moon*; but an *Annulus*, or Ring of Light, will appear round the Border of that Luminary, as happen'd April 1ſt, 1764.

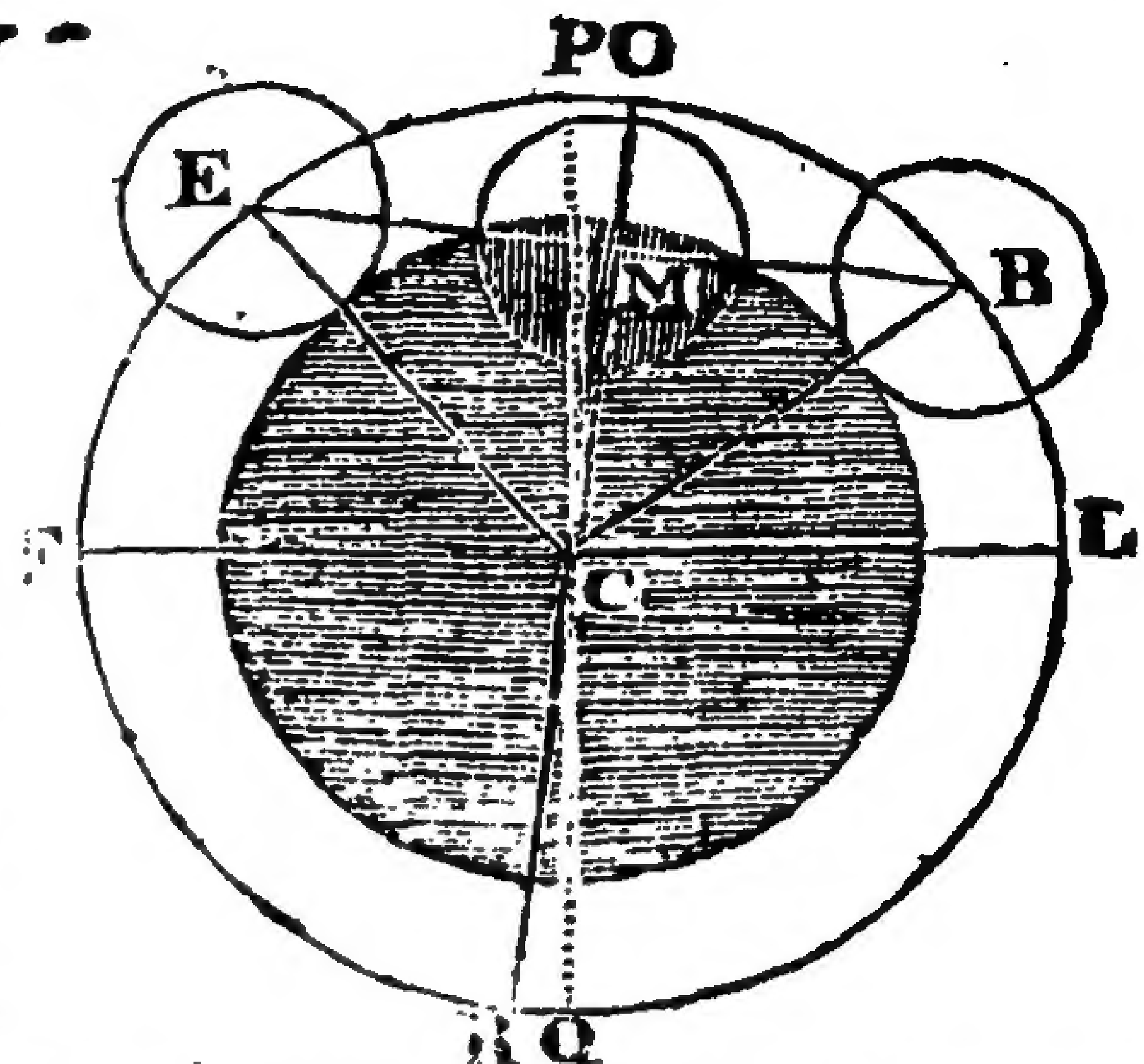
* The Method of finding the *Diſtance* of the *Moon*, in Semidiameters of the Earth, is ſhewn at Problem XVI.

PROBLEM

PROBLEM XXIII.

To calulate the *Beginning*, *End*, and *total Duration* of an *Eclipse*.

Having, from *Astronomical Tables*, obtain'd the *Time* of the *Middle of the Eclipse*, which suppose to be *December* 21 Day, 11 Hours, 49 Minutes, with the *Latitude* of the *Moon* at that Time, = 40 Minutes, you may then proceed to find the *Beginning*, *End*, and *total Duration*, as follows.



From a Scale of equal Parts, of any Size, take off the *Semidiameter* of the *Earth*'s Shadow, which, at the Distance of the ☽ (at a Mean) is about 42′; and, setting one Foot in C, describe the inner *shaded Circle*, to express the Part of the Cone of the Earth's Shadow cut off at that Place where the ☽ passes through in that *Eclipse*.—With the *Sum* of the *Semidiameter* of the ☽ = 16′, and *Earth's* Shadow = 42′, (which together = 58′) taken from the same equal Parts describe the outer Circle.—Draw the Line FL through the Center, to represent the *Ecliptic*, or Path of the *Earth's* Shadow; cross it, at Right Angles, with the dotted Line PQ, to express the *Poles* of the *Ecliptic*.—Then with a *Line of Chords*, or a *Protractor*, set off 5½° from P, upon the outer Circle, towards the *Right Hand*, because the *Latitude* of the ☽ is *North ascending*, to express the Angle of the *Moon's Path* with the *Ecliptic*, and draw the Line OR.—Take the *Latitude* of the ☽ = 40′, from the same Scale of equal Parts, and set it from C to M upon the Line CO.—Then draw a Line through M, at Right Angles to CO, and that Line will represent the *Path* of the ☽ during the *Eclipse*.—Next, with the *Semidiameter* of the ☽ = 16′, taken from the equal Parts, describe, on the three Points B, M, and E, severally, the three *little Circles*; so will the Circle at B represent the ☽ at the *Beginning*, that at M the *Middle*, and that at E, the *End* of the Eclipse.

Now, from the Center C, draw two Lines to B and E; then in the *Right Angled Triangle* CMB, Right Angled at M, we have given CM the *Latitude* of the ☽ = 40, and CB = CE the *Sum* of the *Semidiameters* of the *Moon* and *Earth's Shadow* = 58; to find MB = ME, the *Motion* of *Half Duration* of the *Eclipse*.

From Square of 58 = 3364
Take Square of 40 = 1600

Extract the Root 1764(42 *Minutes*, = the Motion of *Half* the *Duration*. And because the Moon
16 passes over 31 of these Minutes (at a mean Rate) in one Hour, we have
this Proportion.—If 31 Min. : 1 Hour : : 42 Min. : 1 Hour 21 Min.
82)164 which *subtracted from*, and *added to*, the Middle, will give the *Beginning*
164 and *End*.

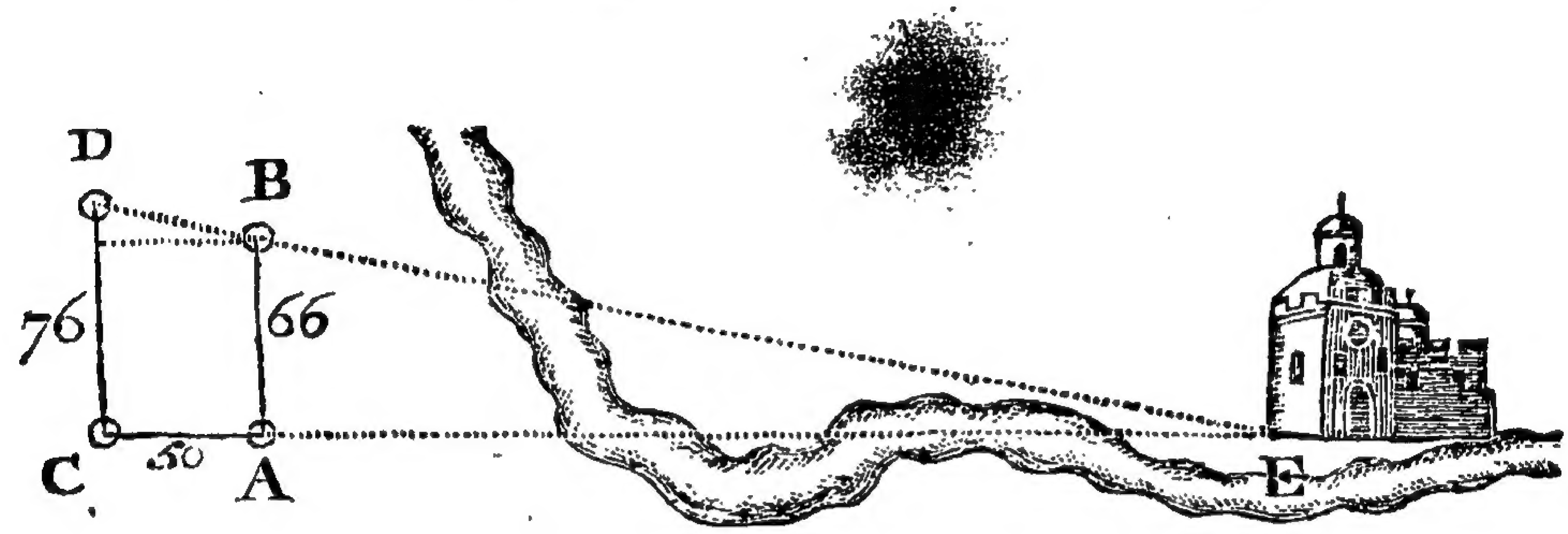
	D.	H.	M.
Middle of the Eclipse	Dec. 21	11	49
Half Duration subtract and add		1	21
Beginning	21	10	28
End	21	13	10
Total Duration		2	42

NOTE. The Middle of an *Eclipse*, with the *Latitude* of the *Moon*, may be easily had from my *Astronomy*.

PROBLEM XXIV.

To take the *Diſtance* of an *inacceſſible Object* without the Help of *any Inſtrument.*

Suppoſe E, in the following Figure, to be a *Fort,* whoſe *Diſtance* you want to know, and you cannot approach it, on Account of ſome *Moat, Ditch,* or *River,* lying between you and the *Object.*



Firſt, at ſome Diſtance from the *Ditch* or *River,* ſet up a *Stick,* as at C; then advance, in a Right Line, towards E, any Number of Yards, ſuppoſe 50, and ſet up another Stick at A; next, move, in a Line *perpendicular* to CE, from A to B, any Diſtance, ſuppoſe 66 Yards, and ſet up another Stick at B; then return back to C, where you began, and remove from thence, in a Line *perpendicular* to CE, till you ſee the Stick at B and the Object E in a Right Line, and ſet up another Stick in that Place at D, meaſuring the Diſtance from C to D, which ſuppoſe 76 Yards. Then it will always hold;—

$$\left.\begin{array}{lcr}\text{As the Difference between AB and CD} & = & 10\\ \text{Is to the Length between A and C} & = & 50\\ \text{So is the Diſtance CD} & = & 76\\ \text{To the Diſtance between C and E} & = & 380\end{array}\right\}\text{Yards.}$$

Note. If, in the *third Term,* you had us'd the *Diſtance* AB = 66 Yards, you would have obtain'd the Diſtance from A to E = 330.

PROBLEM

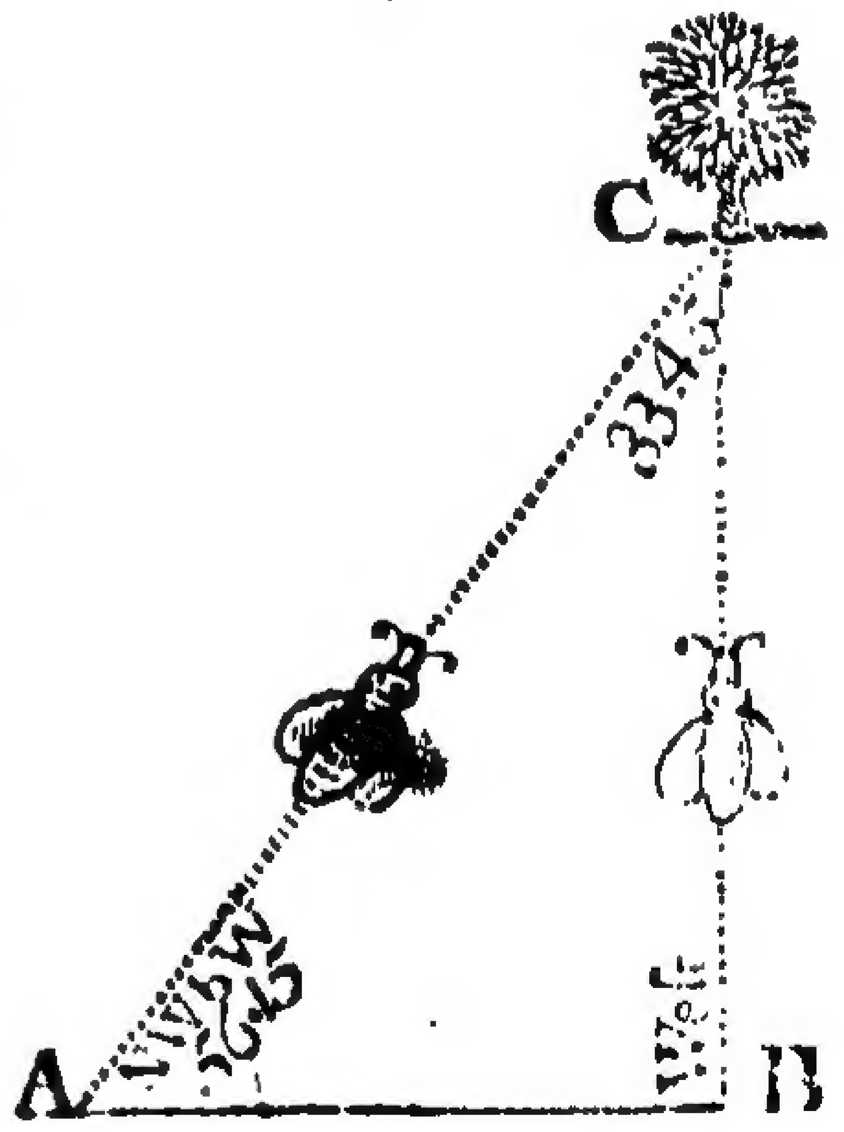
PROBLEM XXV.

To find, by a *new Method*, where the *Bees* hive in large and extensive *Woods*, in Order to obtain their *Honey*.

Take a *Plate* or small Piece of *Board*, on which is spread a little *Honey* or *Treacle*, and set it down on a *Rock* or Stump of a *Tree* within the *Wood*. This the *Bees* will soon find out if any are near, for it is generally believ'd they smell Things of that Nature at the Distance of a Mile, or further. Whilst th[illegible]tle *Creatures* are feeding, secure *two* or *three* of them in a B[illegible]mething convenient. Then let one of them go, observing ca[illegible]y a *Pocket Compass*, the Course he takes; for *Bees*, after they[illegible] the *Air*, fly directly in a strait Line to the *Tree* where their Hive is.

Suppose, for Example, the *first Bee* is found to fly directly *West*; then you may be sure the *Tree* is some where in that Line from your present Station. But, in order to know how far, you must make an *Offset*, either *North* or *South*, as large as you can, which in this case we will suppose to be 100 *Rods* or *Perches* (the larger the better) to the *South*. Here you must let go another *Bee*, observing his Course as before, (for this *Bee*, being loaded like the other, will fly *directly* to the *Hive*) which Course we will suppose to be N. W. by W.—56° 15' towards the *West*, it only remains now to find where these two Courses or Lines intersect or meet with each other, for there you will find the Tree in which the *Honey* is.

This may be easily done.—For in the *Right Angled Triangle* ABC, are given the Right Angle B, the *Course* of the first Bee, the Angle at A, the *Course* of the second Bee, and the Distance AB; to find BC, or AC, the Distance of the *Tree* from either Station.



∠ C	: Base	:: N. Rad.	: Dist. AC
As 33 75	— 100	—— 60.7	— 179.8 Perches.

Formerly, they found the *Honey* by surprizing the Bees, and following them, one after another, till they found out the Hive; [illegible] since this *Trigonometrical Method* has been us'd, the Searchers discover that Booty in a few Hours, which before requir'd many Days.

CONCLUSION.

CONCLUSION.

THESE few *Problems* are ſufficient to point out the great *Uſe* of this Branch of *Learning*. The Advantages reſulting from it to Society are very great;—almoſt infinite.——Nothing however poſited in the *Heavens*;—nothing upon the *Earth* or *Seas*;—but its *Diſtance* and *Dimenſions* may be aſcertained by it.——It is no Wonder then, that *Pythagoras*, a learned Philoſopher of *Samos*, when he had diſcover'd that famous *Propoſition* (47t[illegible] 1ſt Book of *Euclid*) which is the *Foundation* of this *Science*, ſ[illegible]in Gratitude, ſacrifice an *Hecatomb*, i. e. 100 Oxen, to the [illegible]or inſpiring him with ſuch an uſeful Invention, which he judg[illegible]ond the Power of *human Abilities* to diſcover.

Thus by one plain *Geometrical Figure*, having three Sides and three Angles, and aſſiſted by the *Rule of Three*, you ſee what amazing *Truths* may be diſcover'd. This illuſtrates not only the old Motto,----*Tria ſunt omnia* *---but alſo proves the *Truth* of that in the *Title-page*.

Cuncta Trigonus *habet, ſatagit quæ docta* Matheſis,
Ille *aperit clauſum quicquid* Olympus *habet*.

Which may be Engliſh'd thus:

In Heaven *the* latent Science *lay conceal'd*,
Till the Triangle *came, and* Truth *reveal'd*.

* *Omnium prope Deorum Poteſtas triplici Signo oſtenditur; ut, Jovis triſidum Fulmen, Neptuni Tridens, Plutonis Canis triceps; vel quod Omnia ternario continentur.*

FINIS.